Mathematics overview: Stage 6

Unit	*Hours*		
Numbers and the number system	8		
Calculating	8		
Calculating: division	8		
Visualising and constructing	8		
Investigating properties of shapes	8		
Algebraic proficiency: using formulae	4		
Exploring fractions, decimals and percentages	8		
Proportional reasoning	4		
Pattern sniffing	4		
Measuring space	4		
Investigating angles	4		
Calculating fractions, decimals and percentages	12		
Solving equations and inequalities	4		
Calculating space	8		
Checking, approximating and estimating	4		
Mathematical movement	4		
Presentation of data	4		
Measuring data	4		

Numbers and the number system — 8 hours

Key concepts

The Big Picture: Number and Place Value progression map

- identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places
- read, write, order and compare numbers up to 10 000 000 and determine the value of each digit
- use negative numbers in context, and calculate intervals across zero
- identify common factors, common multiples and prime numbers

Return to overview

Possible learning intentions

- Understand and use decimals with up to three decimal places
- Work with numbers up to ten million
- Explore the use of negative numbers
- Develop understanding of factors and multiples
- Investigate prime numbers

Bring on the Maths^{+}: Moving on up!
Number and Place Value: #3, #6
Calculating: #2

Possible success criteria

- Understand place value in numbers with up to three decimal places
- Multiply whole numbers by 10 (100, 1000)
- Divide whole numbers by 10 (100, 1000) when the answer is a whole number
- Multiply (divide) numbers with up to three decimal places by 10 (100, 1000)
- Understand (order, write, read) place value in numbers with up to eight digits
- Understand and use negative numbers when working with temperature
- Understand and use negative numbers when working in other contexts
- Know the meaning of a common multiple (factor) of two numbers
- Identify common multiples (factors) of two numbers
- Know how to test if a number up to 120 is prime

Prerequisites

- Understand and use place value in numbers with up to seven digits
- Multiply and divide whole numbers by 10, 100, 1000
- Multiply and divide numbers with one decimal place by 10, 100, 1000
- Know the meaning of 'factor' and 'multiple' and 'prime'

Bring on the Maths^{+}: Moving on up!
Number and Place Value: #1
Solving problems: #3

Mathematical language

Place value
Digit
Negative number
(Common) multiple
(Common) factor
Divisible
Prime number, Composite number

Pedagogical notes

Zero is neither positive nor negative.
When multiplying and dividing by powers of ten, the decimal point is fixed and it is the digits that move.
Ensure that pupils can deal with large numbers that include zeros in the HTh and/or H column (e.g. 43 006 619)
NCETM: Glossary

Common approaches
The following definition of a prime number should be used in order to minimise confusion about 1: A prime number is a number with exactly two factors.
Every classroom has a set of number classification posters, a place chart and a negative number washing line on the wall.

Reasoning opportunities and probing questions

- Convince me that 109 is a prime number
- Jenny writes 2.54 × 10 = 25.4. Kenny writes 2.54 × 10 = 25.40. who do you agree with? Explain why.
- Look at this number (24 054 028). Show me another number (with 4, 5, 6, 7 digits) that includes a 5 with the same value. And another. And another …

NCETM: Place Value Reasoning

Suggested activities

KM: Maths to Infinity: Directed numbers
KM: Extend the idea of Eratosthenes' sieve to a 12 by 12 grid
KM: Exploring primes activities: Artistic Eratosthenes sieve
KM: Use Powers of ten to demonstrate connections.
NRICH: Factor-multiple chains
NRICH: The Moons of Vuvv
NRICH: Round and round the circle
NRICH: Counting cogs

Learning review
KM: 6M1 BAM Task
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- Some pupils can confuse the language of large (and small) numbers since the prefix 'milli- means 'one thousandth' (meaning that there are 1000 millimetres in a metre for example) while one million is actually a thousand thousand.
- Some pupils may not realise that degrees (°) and degrees Celsius (°C) are two different and distinct units of measurement
- Some pupils may think that 1 is a prime number

Calculating — 8 hours

Key concepts

The Big Picture: Calculation progression map

- perform mental calculations, including with mixed operations and large numbers
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- solve problems involving addition, subtraction and multiplication
- use their knowledge of the order of operations to carry out calculations

Return to overview

Possible learning intentions

- Develop mental calculation skills
- Extend written methods of multiplication
- Know and use the order of operations
- Solve problems involving addition, subtraction and multiplication

Bring on the Maths+: Moving on up!
Calculating: #4
Fractions, decimals & percentages: #6
Solving problems: #2

Possible success criteria

- Combine addition and subtraction when multiplying mentally
- Multiply a two-digit number by a single-digit number mentally
- Add a three-digit number to a two-digit number mentally (when bridging of hundreds is required)
- Multiply a four-digit number by a two-digit number using long multiplication
- Identify when addition, subtraction or multiplication is needed as part of solving multi-step problems
- Explain why addition or subtraction is needed at any point when solving multi-step problems
- Solve multi-step problems involving addition, subtraction and/or multiplication
- Know that addition and subtraction have equal priority
- Know that multiplication and division have equal priority
- Know that multiplication and division take priority over addition and subtraction

Prerequisites

- Recall multiplication facts for multiplication tables up to 12 × 12
- Recall division facts for multiplication tables up to 12 × 12
- Understand the commutativity of multiplication and addition
- Multiply a three-digit number by a two-digit number using short multiplication
- Use column addition and subtraction for numbers with more than four digits

Bring on the Maths+: Moving on up!
Calculating: #1
Solving problems: #1

Mathematical language

Addition
Subtraction
Sum, Total
Difference, Minus, Less
Column addition
Column subtraction
Operation
Multiply, Multiplication, Times, Product
Commutative
Factor
Short multiplication
Long multiplication
Estimate

Pedagogical notes

Note that if not understood fully, BIDMAS can give the wrong answer to a calculation; e.g. 6 – 2 + 3.
The grid method is promoted as a method that aids numerical understanding and later progresses to multiplying algebraic statements.
Use a basic and a scientific calculator to work out 2 + 3 × 5. Why are the answers different?
Later in this stage there is chance to develop and practice these skills with an emphasis on checking, approximating or estimating the answer.
KM: Progression: Addition and Subtraction, Progression: Multiplication and Division and Calculation overview
NCETM: The Bar Model, Subtraction, Multiplication, Multiplicative reasoning, Glossary

Common approaches
All classrooms display a times table poster with a twist
To avoid confusion with language, all teachers use 'sum' to refer only to the result of an addition. Teachers say 'complete these calculations' instead of 'complete these sums'
Long multiplication is promoted as the 'most efficient method'.
If any acronym is promoted to help remember the order of operations, then BIDMAS is used to strengthen progression as the I stands for indices.

Reasoning opportunities and probing questions

- Find missing digits in otherwise completed long multiplication calculations
- Convince me that 2472 × 12 = 29664
- Why have you chosen to add (subtract, multiply)?

NCETM: Addition and Subtraction Reasoning
NCETM: Multiplication and Division Reasoning

Suggested activities

KM: Long multiplication template
KM: Maximise, minimise. Adapt ideas to fit learning intentions.
KM: Maths to Infinity: Complements
KM: Maths to Infinity: Multiplying and dividing
NRICH: Become Maths detectives
NRICH: Exploring number patterns you make
NRICH: Reach 100
Learning review
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- Some pupils may write statements such as 140 - 190 = 50
- When subtracting mentally some pupils may deal with columns separately and not combine correctly; e.g. 180 – 24: 180 – 20 = 160. Taking away 4 will leave 6. So the answer is 166.
- The use of BIDMAS (or BODMAS) can imply that division takes priority over multiplication, and that addition takes priority over subtraction. This can result in incorrect calculations.

Calculating: division — 8 hours

Key concepts

The Big Picture: Calculation progression map

- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division; interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- use written division methods in cases where the answer has up to two decimal places
- solve problems involving division
- use their knowledge of the order of operations to carry out calculations involving the four operations

Return to overview

Possible learning intentions

- Develop written methods of division
- Deal with remainders when carrying out division
- Solve problems involving the four operations

Bring on the Maths^{+}: Moving on up!
Calculating: #5

Possible success criteria

- Use short division to divide a four-digit number by a one-digit number
- Use short division to divide a three- (or four-) digit number by a two-digit number
- Understand the method of long division
- Use long division to find the remainder at each step of the division
- Know how to write, and use, the remainder at each step of the division
- Use long division to divide a three- (or four-) digit number by a two-digit number
- Write the remainder to a division problem as a remainder
- Write the remainder to a division problem as a fraction
- Extend beyond the decimal point to write the remainder as a decimal
- Identify when division is needed to solve a problem
- Extract the correct information from a problem and set up a written division calculation
- Interpret a remainder when carrying out division

Prerequisites

- Use knowledge of multiplication tables when dividing
- Know how to use short division

Mathematical language

Commutative
Divide, Division, Divisible
Divisor, Dividend, Quotient, Remainder
Factor
Short division
Long division
Remainder
Operation
Estimate

Notation
Remainders are often abbreviated to 'r' or 'rem'

Pedagogical notes

Long division is a method of division that formalises how to find a remainder at each step. Short division is a compact method that uses pupils' abilities to find this remainder without the need for a formal written method.
An alternative to both these methods is 'Galley division'.
Later in this stage there is a further opportunity to develop and practice calculation skills with a particular emphasis on checking, approximating or estimating the answer.
KM: Progression: Multiplication and Division and Calculation overview
NCETM: The Bar Model, Division, Glossary

Common approaches
The use of long multiplication is promoted as the 'most efficient method'.
Short division is promoted as the 'most efficient method'.
When dealing with remainders in division problems, use the notation 'r'

Reasoning opportunities and probing questions

- Find missing digits in otherwise completed long / short division calculations
- Show me a calculation that is connected to 147 × 26 = 3822. And another, and another …
- Show me a division calculation that has no remainder. Now show me a division by a two-digit number that has no remainder. And now, a four-digit number divided by a two-digit number that has no remainder. And now, with a remainder of 3 …

NCETM: Multiplication and Division Reasoning

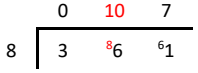
Suggested activities

KM: Dividing (lots)
KM: Maths to Infinity: Multiplying and dividing
KM: Interactive long division
KM: Interactive target boards
KM: Maths to Infinity: Multiplication and division foundations
NRICH: Interactive division
NRICH Dicey Operations. Game 6.

Learning review
KM: 6M2 BAM Task
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- Some pupils may write statements such as 12 ÷ 132 = 11
- Formal written methods of addition, subtraction and multiplication work from right to left. Formal division works from left to right.
- When using short division many pupils will at first struggle to deal correctly with any division where the divisor is greater than the first digit of the dividend; for example:



3 ÷ 8 = 0 remainder 3, and so the 3 should be moved across. Instead, the 8 has been 'moved across' and therefore everything that follows has been correctly carried out based on an early misunderstanding.

Visualising and constructing — *8 hours*

Key concepts

- draw 2-D shapes using given dimensions and angles
- recognise, describe and build simple 3-D shapes, including making nets

The Big Picture: Properties of Shape progression map

Return to overview

Possible learning intentions

- Construct 2D shapes
- Investigate 3D shapes
- Explore nets of 3D shapes

Bring on the Maths^{+}: Moving on up!
Properties of shapes: #1, #2, #3, #4

Possible success criteria

- Use a protractor to draw angles up to 180°
- Use a protractor to work out and construct reflex angles
- Use a ruler to draw lines to the nearest millimetre
- Use squared paper to guide construction of 2D shapes
- Complete tessellations of given shapes
- Know the names of common 3D shapes
- Use mathematical language to describe 3D shapes
- Construct 3D shapes from given nets
- Use 'Polydron' to construct nets for common 3D shapes
- Draw accurate nets for common 3D shapes
- Find all the nets for a cube
- Use a net to visualise the edges (vertices) that will meet when folded

Prerequisites

- Know the names of common 2D shapes
- Know the names of common 3D shapes
- Use a protractor to measure and draw angles

Mathematical language

Protractor
Measure
Nearest
Construct
Sketch
Cube, Cuboid, Cylinder, Pyramid, Prism
Net
Edge, Face, Vertex (Vertices)
Visualise

Notation
Dash notation to represent equal lengths in shapes and geometric diagrams
Right angle notation

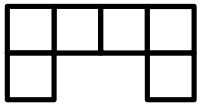
Pedagogical notes

A prism must have a polygonal cross-section, and therefore a cylinder is not a prism. Similarly, a cone is not a pyramid.
A cube is a special case of a cuboid, and a cuboid is a special case of a prism.
Many pupils struggle to sketch 3D shapes. A good strategy for any type of prism is to draw the cross-section (using squares for guidance), and then draw a second identical shape offset from the first. The matching corners can then be joined with straight lines. Some dotted lines (or rubbing out of lines) will be required.
NCETM: Glossary

Common approaches
Every classroom has a set of 3D shape posters on the wall
Models of 3D shapes to be used by all students during this unit of work
All pupils to use 'Polydron' to explore nets of 3D shapes during this unit of work

Reasoning opportunities and probing questions

- Show me an example of a net of a cube. And another. And another …
- What is wrong with this attempt at a net of a cuboid? How can it be changed?



- How many different ways are there to complete these nets?

NCETM: Geometry - Properties of Shapes Reasoning

Suggested activities

KM: Visualising 3D shapes
KM: Tessellating Tess
KM: Fibonacci's disappearing squares
KM: Unravelling dice
NRICH: Making spirals
NRICH: Cut nets
NRICH: Making cuboids

Learning review
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- Some pupils will read the wrong way round the scale on a typical semi-circular protractor, therefore using 180° - required angle
- Some pupils may measure from the end of a ruler, rather than the start of the measuring scale
- Some pupils may think that several repeats of a shape in any pattern constitutes a tessellation
- When given a net of a 3D shape some pupils may think that the number of vertices of the 3D shape is found by counting the number of 'corners' on the net

Investigating properties of shapes	*8 hours*

Key concepts

The Big Picture: Properties of Shape progression map

- compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons
- illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius

Return to overview

Possible learning intentions	Possible success criteria
• Investigate properties of 2D shapes • Investigate angles in polygons • Understand and use the vocabulary of circles **Bring on the Maths^{+}: Moving on up!** Properties of shapes: #5	• Know the definitions of special triangles • Know the definitions of special quadrilaterals • Classify 2D shapes using given categories; e.g. number of sides, symmetry • Know the angle sum of a triangle • Know the angle sum of a quadrilateral • Know how to find the angle sum of a any polygon • Use the angle sum of a triangle to find missing angles • Find the missing angle in an isosceles triangle when only one angle is known • Use the angle sum of a quadrilateral to find missing angles • Know how to find the size of one angle in any regular polygon

Prerequisites	Mathematical language	Pedagogical notes
• Know the properties of rectangles • Know the difference between a regular and an irregular polygon • Add and subtract numbers up to three digits	Quadrilateral, Square, Rectangle, Parallelogram, (Isosceles) Trapezium, Kite, Rhombus, Delta, Arrowhead Triangle, Scalene, Right-angled, Isosceles, Equilateral Polygon, Regular, Irregular Pentagon, Hexagon, Octagon, Decagon, Dodecagon Circle, Radius, Diameter, Circumference, Centre Parallel Diagonal Angle **Notation** Dash notation to represent equal lengths in shapes and geometric diagrams Right angle notation	Ensure that pupils do not use the word 'diamond' to describe a kite, or a square that is 45° to the horizontal. 'Diamond' is not the mathematical name of any shape. A square is a special case of a rectangle. An oblong is a rectangle that is not a square. A rhombus is a special case of a parallelogram. All polygons up to 20 sides have names, although many have alternatives based on either Latin or Greek. Splitting any polygon into triangles (by drawing all diagonals from one vertex) will allow pupils to find the angle sum of the polygon. NCETM: Glossary **Common approaches** *All teachers refer to a 'delta' instead of an 'arrowhead'* *Every classroom has a set of triangle posters and quadrilateral posters on the wall*

Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
• Convince me that a rhombus is a parallelogram • Jenny writes that 'Diameter = 2 × Radius'. Kenny writes that 'Radius = 2 × Diameter'. Who is correct? • What is the same and what is different: a square and a rectangle? NCETM: Geometry - Properties of Shapes Reasoning	KM: Shape work: Many of the activities are suitable for this unit. KM: Dotty activities KM: Investigating polygons. Tasks one and two. KM: Special polygons NRICH: Where Are They? NRICH: Round a Hexagon NRICH: Quadrilaterals KM: 6 point circles, 8 point circles and 12 point circles can be used to support and extend the above idea **Learning review** NCETM: NC Assessment Materials (Teaching and Assessing Mastery)	• Some pupils may think that a 'regular' polygon is a 'normal' polygon • Some pupils may think that all polygons have to be regular • Some pupils may think that a square is only square if 'horizontal', and even that a 'non-horizontal' square is called a diamond • The equal angles of an isosceles triangle are not always the 'base angles' as some pupils may think

Algebraic proficiency: using formulae	*4 hours*
Key concepts • use simple formulae • convert between miles and kilometres	**The Big Picture**: Algebra progression map

Return to overview

Possible learning intentions	Possible success criteria
• Use simple formulae written in words • Create simple formulae written in words • Work with formulae written algebraically **Bring on the Maths^{+}: Moving on up!** Algebra: #1	• Recognise a simple formula written in words • Interpret the information given in a written formula • Substitute numbers into a one-step formula written in words • Substitute numbers into a two-step formula written in words • Interpret the information that results from substituting into a formula • Create a one-step formula from given information • Create a two-step formula from given information • Use symbols to represent variables in a formula

Prerequisites	Mathematical language	Pedagogical notes
• Know the order of operations • Know the fact that area of rectangle = length × width	Formula, Formulae Expression Variable Substitute Symbol Mile Kilometre Metric Imperial **Notation** When written algebraically a formula should not include any units.	Pupils have already used the written formula 'area of rectangle = length × width'. This can be used here to introduce the use of letters to represent variables; 'A = l × w'. Later in the year pupils will meet other formulae for area and volume and this unit should be used to develop conceptual understanding in readiness for this. Other common examples that could be used include the rough conversion between miles and kilometres, 'kilometres = miles × 1.6'. NCETM: Algebra NCETM: Glossary **Common approaches**

Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
• Look at this formula. Write down a fact that it tells you. And another. And another … • Jenny and Kenny are using the formula 'Cost in pounds = 40 + 20 × number of hours' to work out the cost for three hours. Jenny writes down £180. Kenny writes down £100. Who do you agree with? Why? • Always / Sometimes / Never: The formula T = 4n + 6 results in an odd number. NCETM: Algebra Reasoning	KM: Fascinating food NCETM: Year 6 Algebra. Activities A and D. **Learning review** KM: 6M3 BAM Task NCETM: NC Assessment Materials (Teaching and Assessing Mastery)	• Some pupils may apply the order of operations incorrectly when working with two step formulae • Units must be consistent when using formulae. For example, a mobile phone plan might charge £15 per month plus 5p for every text. The formula 'Monthly cost = 15 + 5 × number of texts is wrong because amounts in both pounds and pence are involved. Monthly cost (in pence) = 1500 + 5 × number of texts is one correct way of writing the formula. • It is not advisable to abbreviate the formula 'kilometres = miles × 1.6' using letters. 'm' is the normal abbreviation for metres and 'k' can represent £1000. If 'km' is used it could even be interpreted as 'k × m'.

Exploring fractions, decimals and percentages — 8 hours

Key concepts

The Big Picture: Fractions, decimals and percentages progression map

- use common factors to simplify fractions; use common multiples to express fractions in the same denomination
- compare and order fractions, including fractions > 1
- associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, $^3/_8$]
- recall and use equivalences between simple fractions, decimals and percentages, including in different contexts

Return to overview

Possible learning intentions

- Explore the equivalence between fractions
- Use the equivalence between fractions
- Explore the equivalence between fractions, decimals and percentages

Bring on the Maths$^+$: Moving on up!
Fractions, decimals & percentages: #1, #2

Possible success criteria

- Understand that two fractions can be equivalent
- Identify a common factor of two numbers
- Simplify a fraction
- Write a fraction in its lowest terms
- Confirm that a fraction is written in its lowest terms
- Compare two fractions by considering diagrams
- Compare two fractions by considering equivalent fractions
- Compare two top-heavy fractions
- Understand that a fraction is also a way of representing a division
- Know standard fraction / decimal equivalences (e.g. ½ = 0.5, ¼ = 0.25, $^1/_{10}$ = 0.1)
- Work out the decimal equivalents of fifths, eighths and tenths
- Know standard fraction / decimal / percentage equivalences (e.g. 10%, 25%, 50%, 75%)
- Work out the percentage equivalents of fifths, eighths and tenths
- Use the equivalence between fractions, decimals and percentages when solving problems

Prerequisites

- Understand the concept of a fraction as a proportion
- Understand the concept of equivalent fractions
- Understand the concept of fractions, decimals and percentages being equivalent
- Know that a percentage means 'out of 100'

Mathematical language

Fraction
Improper fraction, Proper fraction, Vulgar fraction, Top-heavy fraction
Percentage
Decimal
Proportion
Simplify
Equivalent
Lowest terms

Notation
Diagonal fraction bar / horizontal fraction bar

Pedagogical notes

Use language carefully to avoid later confusion: when simplifying fractions, the language 'divide by 4' should not be used in place of 'divide the top and bottom by 4'. A fraction can be divided by 4, but that is not the same as cancelling a common factor of the numerator and denominator by dividing them by 4.
NRICH: Teaching fractions with understanding
NCETM: Teaching fractions
NCETM: Glossary

Common approaches
All pupils are made aware that 'per cent' is derived from Latin and means 'out of one hundred'
Teachers use the horizontal fraction bar notation at all times

Reasoning opportunities and probing questions

- Show me another fraction that is equivalent to this one. And another. And another …
- Convince me that 3/8 = 0.375
- If you know that 1/10 = 0.1 = 10%, what else can you work out?
- Jenny is simplifying fractions. She has the fraction 16/64. Jenny says, 'if I cancel out the sixes then 16/64 = 1/4.'. Do you agree with Jenny? Why?

NCETM: Fractions Reasoning

Suggested activities

KM: FDP conversion
KM: Carpets
KM: Fraction and decimal tables
NRICH: Matching fractions
NRICH: Fractions made faster

Learning review
KM: 6M6 BAM Task
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- A fraction can be visualised as divisions of a shape (especially a circle) but some pupils may not recognise that these divisions must be equal in size, or that they can be divisions of any shape.
- Pupils may not make the connection that a percentage is a different way of describing a proportion
- Some pupils may think that simplifying a fraction just requires searching for, and removing, a factor of 2 (repeatedly)

Proportional reasoning — *4 hours*

Key concepts

The Big Picture: Ratio and Proportion progression map

- solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts
- solve problems involving similar shapes where the scale factor is known or can be found
- solve problems involving unequal sharing and grouping using knowledge of fractions and multiples

Return to overview

Possible learning intentions

- Solve problems involving scaling
- Explore enlargement
- Solve problems involving sharing and grouping

Bring on the Maths^{+}: Moving on up!
Ratio and proportion: #1
Y7 Bring on the Maths
Problem Solving: #1, #2, #3

Possible success criteria

- Identify when a comparison problem can be solved using multiplication
- Identify when a comparison problem can be solved using division
- Identify when a comparison problem requires both division and multiplication
- Find the value of a single item in a comparison problem
- Use the value of a single item to solve a comparison problem
- Understand the meaning of enlargement
- Understand the meaning of scale factor
- Recognise when one shape is an enlargement of another
- Use a scale factor to complete an enlargement
- Find the scale factor for a given enlargement
- Use knowledge of fractions to solve a sharing (or grouping) problem
- Use knowledge of multiples to solve a sharing (or grouping) problem

Prerequisites

- Recall multiplication facts for multiplication tables up to 12 × 12
- Recall division facts for multiplication tables up to 12 × 12
- Find fractions of an amount
- Find multiples of a given number

Mathematical language

Proportion
Quantity
Integer
Similar (shapes)
Enlargement
Scale factor
Group
Share
Multiples

Pedagogical notes

Any work on enlargement should only include enlargements using a scale factor. The concept of a centre of enlargement is a future development.
NCETM: The Bar Model
NCETM: Multiplicative reasoning
NCETM: Glossary

Common approaches
All pupils are explicitly taught to use the bar model as a way to represent a problem involving proportional reasoning

Reasoning opportunities and probing questions

- (Given a recipe for 4 people) show me an amount of food that is needed for 8 people, 6 people, 9 people. Show me an amount of food that is needed for a number of people of your choice. And another. And another …
- Convince me that the second shape is an enlargement of the first shape
- Kenny has no sweets. Jenny gives 1/3 of her sweets to Kenny. Jenny now has 18 sweets. Kenny thinks that Jenny had 54 sweets to start with. Kenny is wrong. Explain why.

NCETM: Ratio and Proportion Reasoning

Suggested activities

NRICH: Orange Drink
NRICH: Pumpkin Pie Problem
NRICH: Jumping
NCETM: Activity Set A
NCETM: Activity Set B
NCETM: Activity Set C
NCETM: Activity Set D

Learning review
KM: 6M5 BAM Task
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- Many pupils will want to identify an additive relationship between two quantities that are in proportion and apply this to other quantities in order to find missing amounts
- When finding a fraction of an amount some pupils may try to use a rule formed without the necessary understanding. As a result they will muddle the operations, dividing by the numerator and multiplying by the denominator.
- When constructing an enlargement some pupils may only apply the scale factor in one dimension; for example, 'enlarging' a 2 by 4 rectangle by a scale factor of 2 and drawing a 2 by 8 rectangle.

Pattern sniffing	*4 hours*
Key concepts • generate and describe linear number sequences	**The Big Picture**: Algebra progression map

Return to overview

Possible learning intentions	Possible success criteria
• Explore number sequences **Bring on the Maths^{+}: Moving on up!** Number and Place Value: #4 Number and Place Value: #5	• Use the vocabulary of sequences • Recognise a linear sequence • Describe a number sequence • Find the next term in a linear sequence • Find a missing term in a linear sequence • Generate a linear sequence from its description

Prerequisites	Mathematical language	Pedagogical notes
• Count forwards and backwards in tens (hundreds, thousands) from any positive number up to 10 000 (100 000, 1 000 000) • Count forwards and backwards through zero	Pattern Sequence Linear Term Ascending Descending	Pupils have counted forwards and backwards in previous years and units, but this is the first time that the concept of sequences appears specifically. The language 'term-to-term rule' should not be introduced until Stage 7. NCETM: Algebra NCETM: Glossary **Common approaches** *Teachers and pupils refer to numbers less than zero as 'negative' numbers and not 'minus' numbers*

Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
• Show me a (ascending/descending) linear sequence. And another. And another. • Kenny thinks that 2, 4, 8, 16, … is a linear example. Do you agree? Explain your answer. • Create a linear sequence with a 3rd term of '8'. • Show me a linear sequence where the rule to get from one term to the next is 'add 3'. And another. And another. NCETM: Algebra Reasoning	KM: Maths to Infinity: Sequences NRICH: Times Tables Shifts NRICH: Domino Sets NCETM: Activity B: Sticky Triangles NCETM: Activity D: Generating Sequences **Learning review** KM: 6M4 BAM Task NCETM: NC Assessment Materials (Teaching and Assessing Mastery)	• Some pupils may think linear sequences are only ascending. • Some pupils may think that any sequence that can be described by a rule to get from one term to the next is a linear sequence, e.g. 2, 4, 8, 16, … • Some pupils may not appreciate that both a starting number and a rule to find the next term are required in order to describe a sequence in full.

Measuring space	4 hours

Key concepts

The Big Picture: Measurement and mensuration progression map

- use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places

Return to overview

Possible learning intentions	Possible success criteria
• Solve problems involving measurement **Bring on the Maths+: Moving on up!** Measures: #3	• Convert between non-adjacent metric units; e.g. kilometres and centimetres • Use decimal notation up to three decimal places when converting metric units • Convert between Imperial units; e.g. feet and inches, pounds and ounces, pints and gallons • Solve problems involving converting between measures • State conclusions using the correct notation and units

Prerequisites	Mathematical language	Pedagogical notes
• Convert between adjacent metric units of length, mass and capacity • Know rough equivalents between inches and cm, feet and cm, kg and lb, pint and ml • Use decimal notation to two decimal places when converting between metric unit	Length, distance Mass, weight Volume Capacity Metre, centimetre, millimetre Tonne, kilogram, gram, milligram Litre, millilitre Hour, minute, second Inch, foot, yard Pound, ounce Pint, gallon **Notation** Abbreviations of units in the metric system: m, cm, mm, kg, g, l, ml Abbreviations of units in the Imperial system: lb, oz	Weight and mass are distinct though they are often confused in everyday language. Weight is the force due to gravity, and is calculated as mass multiplied by the acceleration due to gravity. Therefore weight varies due to location while mass is a constant measurement. The prefix 'centi-' means one hundredth, and the prefix 'milli-' means one thousandth. These words are of Latin origin. The prefix 'kilo-' means one thousand. This is Greek in origin. Conversion of volumes will be covered in the 'calculating space' unit. NCETM: Glossary **Common approaches** *Every classroom has a sack of sand (25 kg), a bag of sugar (1 kg), a cheque book (1 cheque is 1 gram), a bottle of water (1 litre, and also 1 kg of water) and a teaspoon (5 ml)*

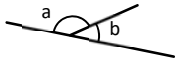
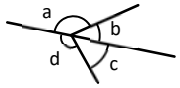
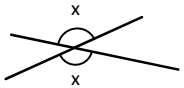
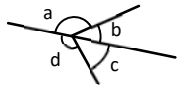
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
• Show me a metric (imperial) unit of measure. And another. And another. • Kenny thinks that 2.5km = 25 000 cm. Do you agree with Kenny? Explain your answer. • Convince me that 4.25kg does not equal 425g. NCETM: Measurement Reasoning	KM: Weighing up the options NRICH: Place Your Orders NRICH: Thousands and Millions NCETM: Activity E : A little bit of history - Marco Polo **Learning review** NCETM: NC Assessment Materials (Teaching and Assessing Mastery)	• Some pupils may apply an incorrect understanding that there are 100 minutes in a hour when solving problems • Some pupils may struggle when converting between 12- and 24-hour clock notation; e.g. thinking that 15:00 is 5 o' clock • Some pupils may apply incorrect beliefs about place value, such as 2.3 × 10 = 2.30. • Many conversions within the metric system rely on multiplying and dividing by 1000. The use of centimetres as an 'extra unit' within the system breaks this pattern. Consequently there is a frequent need to multiply and divide by 10 or 100, and this can cause confusion about the connections that need to be applied.

Investigating angles	*4 hours*
Key concepts • recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles	**The Big Picture**: Position and direction progression map

Return to overview

Possible learning intentions	Possible success criteria
• Develop knowledge of angles • Apply angle facts to deduce unknown angles	• Identify angles that meet at a point • Identify angles that meet at a point on a line • Identify vertically opposite angles • Know that vertically opposite angles are equal • Use known facts to find missing angles • Explain reasoning

Prerequisites	Mathematical language	Pedagogical notes
• Know that angles are measured in degrees • Know that angles in a full turn total 360°, and angle in half a turn must total 180° • Estimate the size of angles	Angle Degrees Right angle Acute angle Obtuse angle Reflex angle Protractor Vertically opposite **Notation** Right angle notation Arc notation for all other angles The degree symbol (°)	The exact reason for there being 360 degrees in a full turn is unknown. There are various theories including it being an approximation of the 365 days in a year and resultant apparent movement of the sun, and the fact that it has so many factors. The SI unit for measuring angles in the radian (2π radians in a full turn). Napoleon experimented with the decimal degree, or grad (400 grads in a full turn) NCETM: Glossary **Common approaches** *All pupils know how to to use a 180° and a 360° protractor.*

Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
• Show a pair of possible values for a and b. And another. And another.  • Convince me that the sum of angles on a straight line is 180°. • Show a possible set of values for a, b, c and d. And another. And another.  • Convince me that the sum of angles around a point is 360°. • Convince me that (vertically) opposite angles are equal. • Kenny thinks that the sum of opposite angles is 180°. Do you agree? Explain your answer. NCETM: Geometry - Properties of Shapes Reasoning	KM: Maths to Infinity: Lines and Angles **Learning review** KM: 6M10 BAM Task NCETM: NC Assessment Materials (Teaching and Assessing Mastery)	• Some pupils may think that these angles are not equal as they are not 'vertical'.  • Some pupils may think that angles that are 'roughly' opposite are always equal, e.g. a = c 

Calculating fractions, decimals and percentages — 12 hours

Key concepts

The Big Picture: Fractions, decimals and percentages progression map

- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $^1/_4 \times {}^1/_2 = {}^1/_8$]
- divide proper fractions by whole numbers [for example, $^1/_3 \div 2 = {}^1/_6$]
- multiply one-digit numbers with up to two decimal places by whole numbers
- solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison

Return to overview

Possible learning intentions

- Calculate with fractions
- Calculate with decimals
- Calculate with percentages

Bring on the Maths$^+$: Moving on up!
Fractions, decimals & percentages: #3, #4, #7
Ratio and proportion: #2

Possible success criteria

- Add (subtract) fractions with different denominators
- Add (subtract) a mixed number and a fraction, including with different denominators
- Add (subtract) mixed numbers, including with different denominators
- Multiply a proper fraction by a proper fraction
- Divide a proper fraction by a whole number
- Simplify the answer to a calculation when appropriate
- Multiply U.t by U
- Multiply U.th by U
- Find 10% of a quantity
- Use non-calculator methods to find a percentage of an amount
- Use decimal or fraction equivalents to find a percentage of an amount where appropriate
- Solve problems involving the use of percentages to make comparisons

Prerequisites

- Convert between mixed numbers and improper fractions
- Find equivalent fractions
- Add and subtract fractions when one denominator is a multiple of the other
- Multiply a proper fraction by a whole number
- Use the formal written method of short multiplication
- Know the effect of multiplying and dividing by 10 and 100
- Know percentage equivalents of $^1/_2$, $^1/_4$, $^3/_4$, $^1/_5$, $^2/_5$, $^4/_5$

Bring on the Maths$^+$: Moving on up!
Fractions, decimals & percentages: #5

Mathematical language

Mixed number
Equivalent fraction
Simplify, cancel
Lowest terms
Proper fraction, improper fraction, top-heavy fraction, vulgar fraction
Numerator, denominator
Percent, percentage

Notation
Mixed number notation
Horizontal / diagonal bar for fractions

Pedagogical notes

Use of a fraction wall to visualise multiplying fractions and dividing fractions by a whole number. For example, pupils need to read calculations such as $\frac{1}{4} \times \frac{1}{2}$ as $\frac{1}{4}$ multiplied by $\frac{1}{2}$ and therefore, $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$; $\frac{4}{10} \div 2$ as $\frac{4}{10}$ divided by 2 and therefore $\frac{2}{10}$.
NCETM: The Bar Model
NCETM: Teaching fractions
NCETM: Fractions videos
NCETM: Glossary

Common approaches
When multiplying a decimal by a whole number pupils are taught to use the corresponding whole number calculation as a general strategy.
When adding and subtracting mixed numbers pupils are taught to convert to improper fractions as a general strategy.
Pupils are encouraged to find and use 10% of an amount.

Reasoning opportunities and probing questions

- Show me an 'easy' ('difficult') pair of fractions to add (subtract). And another. And another.
- Kenny thinks that $\frac{7}{10} - \frac{2}{7} = \frac{5}{3} = 1\frac{2}{3}$. Do you agree with Kenny?
- Jenny thinks that you can only multiply fractions if they have the same common denominator. Do you agree with Jenny? Explain.
- Benny thinks that $\frac{4}{10} \div 2 = \frac{4}{5}$. Do you agree with Kenny? Explain.
- Lenny says *'20% of £60 is £3 because 60 ÷20 = 3'*. Do you agree?

NCETM: Fractions Reasoning
NCETM: Ratio and Proportion Reasoning

Suggested activities

NRICH: Fractions Jigsaw
NRICH: Peaches Today, Peaches Tomorrow
NRICH: Andy's Marbles
NRICH: Would you Rather?

Learning review
KM: 6M7 BAM Task, 6M8 BAM Task, 6M9 BAM Task
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- Some pupils may think that you simply can simply add/subtract the whole number part of mixed numbers and add/subtract the fractional art of mixed numbers when adding/subtracting mixed numbers, e.g. $3\frac{1}{3} - 2\frac{1}{2} = 1\frac{-1}{6}$
- Some pupils may make multiplying fractions over complicated by applying the same process for adding and subtracting of finding common denominators.
- Some pupils may think that as you divide by 10 to find 10%, you divided by 15 to find 15%, divide by 20 to find 20%, etc.

Solving equations and inequalities — 4 hours

Key concepts

- enumerate possibilities of combinations of two variables
- express missing number problems algebraically
- find pairs of numbers that satisfy an equation with two unknowns

The Big Picture: Algebra progression map

Return to overview

Possible learning intentions

- Solve missing number problems
- Understand and use algebra

Bring on the Maths+: Moving on up!
Algebra: #2

Possible success criteria

- Solve missing number problems expressed in words
- Find a solution to a missing number problem with two unknowns
- Find all combinations of two variables that solve a missing number problem with two unknowns
- Know the basic rules of algebraic notation
- Express missing number problems algebraically
- Solve missing number problems expressed algebraically

Prerequisites

- Use symbols to represent variables in a formula

Mathematical language

Algebra, algebraic, algebraically
Symbol
Expression
Variable
Substitute
Equation
Unknown
Enumerate

Notation
The lower case and upper case of a letter should not be used interchangeably when worked with algebra
Juxtaposition is used in place of '×'. $2a$ is used rather than $a2$.
Division is written as a fraction

Pedagogical notes

The word 'algebra' comes from the title of a book by the Persian mathematician, al-Khwārizmī, who lived in modern-day Baghdad about 1200 years ago. Al-kitāb al-mukhtaṣar fī ḥisāb **al-ğabr** wa'l-muqābala was a book that promoted the idea of solving equations by a method of balancing.
Avoid fruit salad algebra (see possible misconceptions).
NCETM: The Bar Model
NCETM: Algebra
NCETM: Glossary

Common approaches
Use 'think of a number' problems to introduce the idea of 'n' standing for any number (a variable). For example:

- *Think of a number, double it, add 8, halve it and finally subtract your starting number.*
  The answer is always 4. $n \rightarrow 2n \rightarrow 2n + 8 \rightarrow n + 4 \rightarrow 4$.

Encourage students to try again with different numbers, including large, small, negative, fractional or decimal. Also try varying the instructions to end up with a different number every time.

Reasoning opportunities and probing questions

- $a + b = 15$. Show me a pair of values for a and b. And another. And another.
- $p + q = 7$. Show me a pair of values for p and q that no one else will think of. And another. And another.
- Kenny thinks that 'b^2 ' is the same as '$2b$' because when $b = 2$, $b^2 = 4$ and $2b = 4$. Do you agree with Kenny? Explain your answer.
- Jenny thinks that $7 + 2a = 9a$. Do you agree with Jenny? Explain your answer.

NCETM: Algebra Reasoning

Suggested activities

NRICH: Plenty of Pens
NRICH: Your Number Is…
NRICH: Number Pyramids
NCETM: Activity A: Racetrack and Design a board game
NCETM: Activity E: Matchbox Algebra

Learning review
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- Some pupils may think that variables have a set value, such as $a = 1$, $b = 2$, $c = 3$, $d = 4$, etc. (especially if they have done lots of poorly designed treasure hunts/codes) – this will lead to problems such as thinking 'b^2 ' is the same as '$2b$' because when $b = 2$, $b^2 = 4$ and $2b = 4$.
- Using the idea of 'apples' and 'bananas' to explain $a + b = 14$ can lead to misconceptions about the use of letters as variables.
- Some students may think that the variables have to be positive integers (whole numbers)

Calculating space — 8 hours

Key concepts

The Big Picture: Measurement and mensuration progression map

- recognise that shapes with the same areas can have different perimeters and vice versa
- calculate the area of parallelograms and triangles
- calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm^3) and cubic metres (m^3), and extending to other units [for example, mm^3 and km^3]
- recognise when it is possible to use formulae for area and volume of shape
- solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate

Return to overview

Possible learning intentions

- Explore area
- Investigate volume
- Solve problems involving area and volume

Bring on the Maths$^+$: Moving on up!
Measures: #6

Possible success criteria

- Recognise that shapes with the same areas can have different perimeters and vice versa
- Know that the area of a parallelogram is given by the formula area = base × height
- Know that the area of a triangle is given by the formula area = ½ × base × height = base × height ÷ 2 = $\frac{bh}{2}$
- Know that the volume of a cuboid is given by the formula volume = length × width × height
- Calculate the area of a parallelogram (triangle)
- Recognise when it is possible to use a formula for the area of a shape
- Estimate the volume of cubes and cuboids
- Choose appropriate units of volume
- Calculate the volume of a cuboid
- Recognise when it is possible to use a formula for the volume of a shape
- Convert between metric units of area in simple cases
- Convert between metric units of volume in simple cases

Prerequisites

- Know the meaning of perimeter (area, volume, capacity)
- Know that the area of a rectangle is given by the formula area = length × width
- Know that area can be measured using square centimetres or square metres, and the abbreviations cm^2 and m^2
- Know that volume is measured in cubes

Bring on the Maths$^+$: Moving on up!
Measures: #4, #5

Mathematical language

Perimeter, area, volume, capacity
Square, rectangle, parallelogram, triangle
Composite rectilinear
Polygon
Cube, cuboid
Millimetre, Centimetre, Metre, Kilometre
Square millimetre, square centimetre, square metre, square kilometre
Cubic centimetre, centimetre cube
Formula, formulae
Convert
Length, breadth, depth, height, width

Notation
Abbreviations of units in the metric system: km, m, cm, mm, mm^2, cm^2, m^2, km^2, mm^3, cm^3, km^3

Pedagogical notes

In this unit, 'volumes of shapes' refers only to cubes and cuboids.
Ensure that pupils make connections with the area of a rectangle work in Stage 5, in particular the importance of the perpendicular height.
Note that there are several different ways of stating the area of a triangle and this can cause confusion
NCETM: Glossary

Common approaches
Pupils derive practically the formulae for area of parallelogram and triangle by dissecting rectangles
Pupils derive the formula for the area of a parallelogram first. They then use this to help derive the formula for the area of an obtuse-angled triangle.
Every classroom has a set of area posters on the wall
Pupils use the area of a triangle as given by the formula area = $\frac{bh}{2}$

Reasoning opportunities and probing questions

- 'Show me' an example of when you would measure volume using km^3
- Convince me that the area of a parallelogram is found using base × height
- (Given a triangle with base labelled 8 cm, height 5 cm, slope height 6 cm) Kenny thinks that the area is 40 cm^2, Lenny thinks it is 20 cm^2, Jenny thinks it is 240 cm^2 and Benny thinks it is 24 cm^2. Who do you agree with? Explain why.

NCETM: Geometry -Properties of Shapes Reasoning

Suggested activities

KM: Fibonacci's disappearing squares
KM: Dissections deductions
KM: Stick on the Maths SSM9: Area and volume
KM: Maths to Infinity Area and Volume
NCETM: Activity C: Through the window

Learning review
KM: 6M11 BAM Task
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- Some pupils may use the sloping height when finding the areas of parallelograms and triangles
- Some pupils may think that the area of a triangle is found using area = base × height
- Some pupils may think that you multiply all the numbers to find the area of a shape

Checking, approximating and estimating — 4 hours

Key concepts

The Big Picture: Number and Place Value progression map

- solve problems which require answers to be rounded to specified degrees of accuracy
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy
- round any whole number to a required degree of accuracy

Return to overview

Possible learning intentions

- Explore ways of approximating numbers
- Explore ways of checking answers

Possible success criteria

- Approximate any number by rounding to the nearest 1 000 000
- Approximate any number by rounding to a specified degree of accuracy; e.g. nearest 20, 50
- Understand estimating as the process of finding a rough value of an answer or calculation
- Use estimation to predict the order of magnitude of the solution to a (decimal) calculation
- Check the order of magnitude of the solution to a (decimal) calculation
- Estimate multiplication calculations that involve multiplying up to four-digit numbers by a two-digit number
- Estimate division calculations that involve dividing up to a four-digit number by a two-digit number
- Estimate multiplication calculations that involve multiplying numbers with up to two decimal places by whole numbers

Prerequisites

- Approximate any number by rounding to the nearest 10, 100 or 1000, 10 000 or 100 000
- Approximate any number with one or two decimal places by rounding to the nearest whole number
- Approximate any number with two decimal places by rounding to the one decimal place
- Estimate addition (subtraction) calculations with up to four digits

Mathematical language

Approximate (noun and verb)
Round
Decimal place
Check
Solution
Answer
Estimate (noun and verb)
Order of magnitude
Accurate
Accuracy

Notation
The approximately equal symbol (≈)

Pedagogical notes

This unit is an opportunity to develop and practice calculation skills with a particular emphasis on checking, approximating or estimating the answer.
Pupils should use numbers up to 10 000 000 in this unit.
Pupils should be able to round to other specified degrees of accuracy, but not to a specified number of significant figures, which is introduced in Stage 7.
Also see big pictures: Calculation progression map and Fractions, decimals and percentages progression map
NCETM: Glossary

Common approaches
All pupils are taught to visualise rounding through the use a number line

Reasoning opportunities and probing questions

- Convince me that 67 rounds to 60 to the nearest 20
- Convince me that 1 579 234 rounds to 2 million to the nearest million
- Jenny writes 1359 ÷ 18 ≈ 7.55. Comment on Jenny's approximation.
- Lenny writes 2.74 × 13 ≈ 26. Do you agree with Lenny? Explain your answer.

NCETM: Place Value Reasoning

Suggested activities

KM: Stick on the Maths CALC6: Checking results
KM: Maths to Infinity Rounding
NRICH: Four Go
NCETM: Activity A(i)
NCETM: Activity G

Learning review
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- Some pupils may truncate instead of round
- When checking the order of magnitude of a division calculation some pupils may apply incorrect reasoning about the effect of increasing the divisor by a factor of 10, thinking that it also makes the solution greater by a factor of 10; e.g. 1400 ÷ 20: 1400 ÷ 2 = 700 so 1400 ÷ 20 = 7000.
- Some pupils may round down at the half way point, rather than round up.

Mathematical movement — 4 hours

Key concepts

- describe positions on the full coordinate grid (all four quadrants)
- draw and translate simple shapes on the coordinate plane, and reflect them in the axes

The Big Picture: Position and direction progression map

Return to overview

Possible learning intentions

- Understand and use Cartesian coordinates
- Use transformations to move shapes

Possible success criteria

- Use coordinates to describe the position of a point in all four quadrants
- Use coordinates to write the position of a point in all four quadrants
- Construct a 2-D coordinate grid (all four quadrants)
- Use coordinates to plot the position of a point in any of the four quadrants
- Use coordinates to plot a set of points to construct a polygon
- Solve problems involving coordinates
- Carry out a translation
- Carry out a reflection using one of the axes as a mirror line

Prerequisites

- Use coordinates in the first quadrant
- Identify a translation
- Carry out a translation in the first quadrant
- Identify a reflection
- Carry out a reflection in the first quadrant using mirror lines parallel to the axes
- Know the meaning of 'congruent', 'congruence', 'object', 'image'

Bring on the Maths$^+$: Moving on up!
Properties of shapes: #2
Position and direction: #1

Mathematical language

2-D
Grid
Axis, axes, x-axis, y-axis
Origin
Quadrant
(Cartesian) coordinates
Point
Translation
Reflection
Transformation
Object, Image
Congruent, congruence

Notation
Cartesian coordinates should be separated by a comma and enclosed in brackets (x, y)

Pedagogical notes

The main focus of this unit is to develop understanding of coordinates in all four quadrants
Note that pupils are not yet expected to use an algebraic description of a mirror line (such as x = 3).
The French mathematician Rene Descartes introduced Cartesian coordinates in the 17th century. It is said that he thought of the idea while watching a fly moving around on his bedroom ceiling.
Other coordinate systems include grid references, polar coordinates and spherical coordinates.
There are other types of mathematical movement that pupils will learn about in future stages. The group name for these movements is 'transformations'.
NCETM: Glossary

Common approaches
Teachers do not use the phrase 'along the corridor and up the stairs' as it can encourage a mentality of only working in the first quadrant. Later, pupils will have to use coordinates in all four quadrants. A more helpful way to remember the order of coordinates is 'x is a cross, wise up!'
Teachers use the language 'negative number', and not 'minus number', to avoid future confusion with calculations.

Reasoning opportunities and probing questions

- (Given a grid with the point (-3, 4) indicated) Benny describes this point as (-3, 4). Jenny describes the point as (4, -3). Who do you agree with? Why?
- Two vertices of a rectangle are (-1, 2) and (4, -2). What could the other two vertices be? How many solutions can you find?
- Convince me that (-2, 3) is in the second quadrant)

NCETM: Geometry: Position Direction and Movement Reasoning

Suggested activities

KM: Stick on the Maths ALG2: Coordinates in four quadrants
NRICH: Cops and Robbers
NRICH: Eight Hidden Squares
NRICH: Coordinate Tan
NRICH: Transformation Tease
NCETM: Activity B - Battleships

Learning review
KM: 6M12 BAM Task
NCETM: NC Assessment Materials (Teaching and Assessing Mastery)

Possible misconceptions

- When describing or carrying out a translation, some pupils may count the squares between the two shapes rather than the squares that describe the movement between the two shapes.
- When reflecting a triangle some students may draw a translation
- When carrying out a reflection some pupils may think that the object and image should be an equal distance from the edge of the grid, rather than an equal distance form the mirror line.
- Some pupils will confuse the order of x-coordinates and y-coordinates
- When constructing axes, some pupils may not realise the importance of equal divisions on the axes

Presentation of data	4 hours
Key concepts • interpret and construct pie charts and line graphs and use these to solve problems	**The Big Picture**: Statistics progression map

Return to overview

Possible learning intentions	Possible success criteria
• Construct and interpret pie charts • Solve problems involving graphs and charts **Bring on the Maths+: Moving on up!** Statistics: #2, #3	• Understand that pie charts are used to show proportions • Make statements about proportions shown in a pie charts • Make statements to compare proportions in pie charts • Use additional information to make statements about frequencies in pie charts • Use a table of frequencies to work out the angle for a slice in a pie chart • Construct a pie chart by measuring angles • Identify the scale used on the axes of a graph • Read values from a line graph involving scaling • Use scaling when constructing line graphs • Answer two-step questions about data in line graphs (e.g. 'How much more?')

Prerequisites	Mathematical language	Pedagogical notes
• Measure and construct angles using a protractor • Interpret and construct a simple line graph	Data Scale Axis, axes Graph Frequency Time graph, Time series Line graph Pie chart Sector Angle Protractor Degrees Maximum, minimum	In Stage 6, when constructing pie charts the total of the frequencies is always a factor of 360 More complex cases are included in later stages. William Playfair, a Scottish engineer and economist, introduced the line graph in 1786. He also introduced the pie chart in 1801. NCETM: Glossary **Common approaches** *Pie charts are constructed by calculating the angle for each section by dividing 360 by the total frequency, and not using percentages.* *The angle for the first section is measured from a vertical radius.* *Subsequent sections are measured using the boundary line of the previous section.*

Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
• Show me a pie chart representing the following information: Blue (25%), Red (over 50%), Yellow (the rest). And another. And another. • Always / Sometimes / Never: Pie charts are constructed in a clockwise direction • Always / Sometimes / Never: The larger the size of the pie chart, the greater the total frequency • Kenny says 'If two pie charts have the same section then the amount of data the section represents is the same in each pie chart.' Do you agree with Kenny? Explain your answer. NCETM: Statistics Reasoning	KM: Stick on the Maths HD6: Graphs and diagrams NRICH: Match the Matches NRICH: Graphing Number Patterns NCETM: A little bit of history (Britain since 1945) **Learning review** NCETM: NC Assessment Materials (Teaching and Assessing Mastery)	• Some pupils may think that a line graph is appropriate for discrete data • Some pupils may think that each square on the grid used represents one unit • Some pupils may confuse the fact that the sections of the pie chart total 100% and 360°

Measuring data	4 hours
Key concepts • calculate and interpret the mean as an average	**The Big Picture**: Statistics progression map

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Possible learning intentions	Possible success criteria
• Understand and use the mean **Bring on the Maths^{+}: Moving on up!** Statistics: #4	• Understand the meaning of 'average' as a typicality (or location) • Understand the mean as a measure of typicality (or location) • Interpret the mean as a way of levelling the data • Calculate the mean of a set of data • Choose an appropriate approximation when required • Use the mean to find a missing number in a set of data

Prerequisites	Mathematical language	Pedagogical notes
• Approximate a number by rounding to a given number of decimal places	Average Mean Measure Data Statistic Statistics Approximate Round	The word 'average' is often used synonymously with the mean, but it is only one type of average. In fact, there are several different types of mean (the one in this unit properly being named as the 'arithmetic mean'). Other types of average, including the mode and the median, are introduced in later stages. NCETM: Glossary **Common approaches** *Always use brackets when writing out the calculation for a mean, e.g. (2 + 3 + 4 + 5) ÷ 4 = 14 ÷ 4 = 3.5*

Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
• Always / Sometimes / Never: The mean is a whole number. • Kenny is working out the mean of 2, 3, 4 and 5. He calculates 2 + 3 + 4 + 5 ÷ 4 = 10.25. Do you agree with Kenny? Explain your answer. • The average number of children per family (Married Couples, 2012) is 1.8. Convince me that this statement makes sense. NCETM: Statistics Reasoning	KM: Maths to Infinity: Averages, Charts and Tables NRICH: Birdwatch NRICH: Probably … NRICH: Same or Different? NCETM: A little bit of history (Britain since 1945) **Learning review** KM: 6M13 BAM Task NCETM: NC Assessment Materials (Teaching and Assessing Mastery)	• If using a calculator some pupils may not use the '=' symbol (or brackets) correctly; e.g. working out the mean of 2, 3, 4 and 5 as 2 + 3 + 4 + 5 ÷ 4 = 10.25. • Some pupils may think the average is always the middle number • Some pupils may think that the mean must be a whole number • Some pupils may not realise that the mean must lie within the range of the data set.