

## Mathematics overview: Stage 7

<i>Unit</i>	<i>Hours</i>		
Numbers and the number system	9		
Counting and comparing	4		
Calculating	9		
Visualising and constructing	5		
Investigating properties of shapes	6		
Algebraic proficiency: tinkering	9		
Exploring fractions, decimals and percentages	3		
Proportional reasoning	4		
Pattern sniffing	3		
Measuring space	5		
Investigating angles	3		
Calculating fractions, decimals and percentages	12		
Solving equations and inequalities	6		
Calculating space	6		
Checking, approximating and estimating	2		
Mathematical movement	8		
Presentation of data	6		
Measuring data	5		



**Key concepts**

The Big Picture: [Number and Place Value progression map](#)

- use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor and lowest common multiple
- use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5
- recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions

[Return to overview](#)

Possible learning intentions		Possible success criteria
<ul style="list-style-type: none"> <li>• Solve problems involving prime numbers</li> <li>• Use highest common factors to solve problems</li> <li>• Use lowest common multiples to solve problems</li> <li>• Explore powers and roots</li> <li>• Investigate number patterns</li> </ul>		<ul style="list-style-type: none"> <li>• Recall prime numbers up to 50</li> <li>• Know how to test if a number up to 150 is prime</li> <li>• Know the meaning of 'highest common factor' and 'lowest common multiple'</li> <li>• Recognise when a problem involves using the highest common factor of two numbers</li> <li>• Recognise when a problem involves using the lowest common multiple of two numbers</li> <li>• Understand the use of notation for powers</li> <li>• Know the meaning of the square root symbol (<math>\sqrt{\quad}</math>)</li> <li>• Use a scientific calculator to calculate powers and roots</li> <li>• Make the connection between squares and square roots (and cubes and cube roots)</li> <li>• Identify the first 10 triangular numbers</li> <li>• Recall the first 15 square numbers</li> <li>• Recall the first 5 cube numbers</li> <li>• Use linear number patterns to solve problems</li> </ul>
Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> <li>• Know how to find common multiples of two given numbers</li> <li>• Know how to find common factors of two given numbers</li> <li>• Recall multiplication facts to <math>12 \times 12</math> and associated division facts</li> </ul> <p><b>Bring on the Maths': Moving on up!</b> Number and Place Value: #6</p>	((Lowest) common) multiple and LCM ((Highest) common) factor and HCF Power (Square and cube) root Triangular number, Square number, Cube number, Prime number Linear sequence  <b>Notation</b> Index notation: e.g. $5^3$ is read as '5 to the power of 3' and means '3 lots of 5 multiplied together' Radical notation: e.g. $\sqrt{49}$ is generally read as 'the square root of 49' and means 'the positive square root of 49'; $\sqrt[3]{8}$ means 'the cube root of 8'	Pupils need to know how to use a scientific calculator to work out powers and roots. Note that while the square root symbol ( $\sqrt{\quad}$ ) refers to the positive square root of a number, every positive number has a negative square root too. NCETM: <a href="#">Departmental workshop: Index Numbers</a> NCETM: <a href="#">Glossary</a>  <b>Common approaches</b> <i>The following definition of a prime number should be used in order to minimise confusion about 1: A prime number is a number with exactly two factors.</i> Every classroom has a set of <a href="#">number classification posters</a> on the wall
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
<ul style="list-style-type: none"> <li>• When using Eratosthenes sieve to identify prime numbers, why is there no need to go further than the multiples of 7? If this method was extended to test prime numbers up to 200, how far would you need to go? Convince me.</li> <li>• Kenny says '20 is a square number because <math>10^2 = 20</math>'. Explain why Kenny is wrong. Kenny is partially correct. How could he change his statement so that it is fully correct?</li> <li>• Always / Sometimes / Never: The lowest common multiple of two numbers is found by multiplying the two numbers together.</li> </ul>	KM: <a href="#">Exploring primes activities</a> ; Factors of square numbers; Mersenne primes; LCM sequence; $n^2$ and $(n + 1)^2$ ; $n^2$ and $n^2 + n$ ; $n^2 + 1$ ; $n! + 1$ ; $n! - 1$ ; $x^2 + x + 41$ KM: Use the method of <a href="#">Eratosthenes' sieve</a> to identify prime numbers, but on a grid 6 across by 17 down instead. What do you notice? KM: <a href="#">Square number puzzle</a> KM: <a href="#">History and Culture: Goldbach's Conjectures</a> NRICH: <a href="#">Factors and multiples</a> NRICH: <a href="#">Powers and roots</a>  <b>Learning review</b> KM: <a href="#">7M1 BAM Task</a>	<ul style="list-style-type: none"> <li>• Many pupils believe that 1 is a prime number – a misconception which can arise if the definition is taken as 'a number which is divisible by itself and 1'</li> <li>• A common misconception is to believe that <math>5^3 = 5 \times 3 = 15</math></li> <li>• See pedagogical note about the square root symbol too</li> </ul>



## Key concepts

- order positive and negative integers, decimals and fractions
- use the symbols =, ≠, <, >, ≤, ≥

The Big Picture: [Number and Place Value progression map](#)[Return to overview](#)

Possible learning intentions		Possible success criteria
<ul style="list-style-type: none"> <li>• Compare numbers</li> <li>• Order numbers</li> </ul>		<ul style="list-style-type: none"> <li>• Place a set of negative numbers in order</li> <li>• Place a set of mixed positive and negative numbers in order</li> <li>• Identify a common denominator that can be used to order a set of fractions</li> <li>• Order fractions where the denominators are not multiples of each other</li> <li>• Order a set of numbers including a mixture of fractions, decimals and negative numbers</li> <li>• Use inequality symbols to compare numbers</li> <li>• Make correct use of the symbols = and ≠</li> </ul>
Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> <li>• Understand that negative numbers are numbers less than zero</li> <li>• Order a set of decimals with a mixed number of decimal places (up to a maximum of three)</li> <li>• Order fractions where the denominators are multiples of each other</li> <li>• Order fractions where the numerator is greater than 1</li> <li>• Know how to simplify a fraction by cancelling common factors</li> </ul>	Positive number Negative number Integer Numerator Denominator  <b>Notation</b> The 'equals' sign: = The 'not equal' sign: ≠ The inequality symbols: < (less than), > (greater than), ≤ (less than or equal to), ≥ (more than or equal to)	Zero is neither positive nor negative. The set of integers includes the natural numbers {1, 2, 3, ...}, zero (0) and the 'opposite' of the natural numbers {-1, -2, -3, ...}. Pupil must use language correctly to avoid reinforcing misconceptions: for example, 0.45 should never be read as 'zero point forty-five'; $5 > 3$ should be read as 'five is greater than 3', not '5 is bigger than 3'. Ensure that pupils read information carefully and check whether the required order is smallest first or greatest first. The equals sign was designed by Robert Recorde in 1557 who also introduced the plus (+) and minus (-) symbols. NCETM: <a href="#">Glossary</a>  <b>Common approaches</b> <i>Teachers use the language 'negative number' to avoid future confusion with calculation that can result by using 'minus number'</i> <i>Every classroom has a <a href="#">negative number washing line</a> on the wall</i>
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
<ul style="list-style-type: none"> <li>• Jenny writes down <math>0.400 &gt; 0.58</math>. Kenny writes down <math>0.400 &lt; 0.58</math>. Who do you agree with? Explain your answer.</li> <li>• Find a fraction which is greater than <math>\frac{3}{5}</math> and less than <math>\frac{7}{8}</math>. And another. And another ...</li> <li>• Convince me that <math>-15 &lt; -3</math></li> </ul>	KM: <a href="#">Farey Sequences</a> KM: <a href="#">Decimal ordering cards 2</a> KM: <a href="#">Maths to Infinity: Fractions, decimals and percentages</a> KM: <a href="#">Maths to Infinity: Directed numbers</a> NRICH: <a href="#">Greater than or less than?</a>  <b>Learning review</b> <a href="http://www.diagnosticquestions.com">www.diagnosticquestions.com</a>	<ul style="list-style-type: none"> <li>• Some pupils may believe that 0.400 is greater than 0.58</li> <li>• Pupils may believe, incorrectly, that:               <ul style="list-style-type: none"> <li>- A fraction with a larger denominator is a larger fraction</li> <li>- A fraction with a larger numerator is a larger fraction</li> <li>- A fraction involving larger numbers is a larger fraction</li> </ul> </li> <li>• Some pupils may believe that -6 is greater than -3. For this reason ensure pupils avoid saying 'bigger than'</li> </ul>



**Key concepts**

The Big Picture: [Calculation progression map](#)

- understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)
- apply the four operations, including formal written methods, to integers and decimals
- use conventional notation for priority of operations, including brackets
- recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions)

[Return to overview](#)

**Possible learning intentions**

- Apply understanding of place value
- Explore written methods of calculation
- Calculate with decimals
- Know and apply the correct order of operations

**Possible success criteria**

- Use knowledge of place value to multiply with decimals
- Use knowledge of place value to divide a decimal
- Use knowledge of place value to divide by a decimal
- Use knowledge of inverse operations when dividing with decimals
- Be fluent at multiplying a three-digit or a two-digit number by a two-digit number
- Be fluent when using the method of short division
- Know the order of operations for the four operations
- Use brackets in problem involving the order of operations
- Understand and apply the fact that addition and subtraction have equal priority
- Understand and apply the fact that multiplication and division have equal priority

**Prerequisites**

- Fluently recall multiplication facts up to  $12 \times 12$
- Fluently apply multiplication facts when carrying out division
- Know the formal written method of long multiplication
- Know the formal written method of short division
- Know the formal written method of long division
- Convert between an improper fraction and a mixed number

**Bring on the Maths<sup>+</sup>: Moving on up!**

Calculating: #2, #3, #4, #5

Fractions, decimals & percentages: #6, #7

Solving problems: #2

**Mathematical language**

Improper fraction  
 Top-heavy fraction  
 Mixed number  
 Operation  
 Inverse  
 Long multiplication  
 Short division  
 Long division  
 Remainder

**Pedagogical notes**

Note that if not understood fully, BIDMAS can give the wrong answer to a calculation; e.g.  $6 - 2 + 3$ .

The grid method is promoted as a method that aids numerical understanding and later progresses to multiplying algebraic statements. Later in this stage there is chance to develop and practice these skills with an emphasis on checking, approximating or estimating the answer.

KM: [Progression: Addition and Subtraction](#), [Progression: Multiplication and Division](#) and [Calculation overview](#)

NCETM: [Departmental workshop: Place Value](#)

NCETM: [Subtraction](#), [Multiplication](#), [Division](#), [Glossary](#)

**Common approaches**

*All classrooms display a [times table poster with a twist](#)*

*The use of long multiplication is to be promoted as the 'most efficient method'.*

*Short division is promoted as the 'most efficient method'.*

*If any acronym is promoted to help remember the order of operations, then BIDMAS is used as the I stands for indices.*

**Reasoning opportunities and probing questions**

- Jenny says that  $2 + 3 \times 5 = 25$ . Kenny says that  $2 + 3 \times 5 = 17$ . Who is correct? How do you know?
- Find missing digits in otherwise completed long multiplication / short division calculations
- Show me a calculation that is connected to  $14 \times 26 = 364$ . And another. And another ...

**Suggested activities**

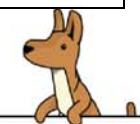
- KM: [Long multiplication template](#)
- KM: [Dividing \(lots\)](#)
- KM: [Interactive long division](#)
- KM: [Misplaced points](#)
- KM: [Maths to Infinity: Multiplying and dividing](#)
- NRICH: [Cinema Problem](#)
- NRICH: [Funny factorisation](#)
- NRICH: [Skeleton](#)
- NRICH: [Long multiplication](#)

**Learning review**

KM: [7M2 BAM Task](#)

**Possible misconceptions**

- The use of BIDMAS (or BODMAS) can imply that division takes priority over multiplication, and that addition takes priority over subtraction. This can result in incorrect calculations.
- Pupils may incorrectly apply place value when dividing by a decimal for example by making the answer 10 times bigger when it should be 10 times smaller.
- Some pupils may have inefficient methods for multiplying and dividing numbers.



**Key concepts**

The Big Picture: [Properties of Shape progression map](#)

- use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries
- use the standard conventions for labelling and referring to the sides and angles of triangles
- draw diagrams from written description

[Return to overview](#)

Possible learning intentions		Possible success criteria
<ul style="list-style-type: none"> <li>• Interpret geometrical conventions and notation</li> <li>• Apply geometrical conventions and notation</li> </ul> <p><b>Bring on the Maths*: Moving on up!</b> Properties of shapes: #3, #4</p>		<ul style="list-style-type: none"> <li>• Know the meaning of faces, edges and vertices</li> <li>• Use notation for parallel lines</li> <li>• Know the meaning of 'perpendicular' and identify perpendicular lines</li> <li>• Know the meaning of 'regular' polygons</li> <li>• Identify line and rotational symmetry in polygons</li> <li>• Use AB notation for describing lengths</li> <li>• Use <math>\angle ABC</math> notation for describing angles</li> <li>• Use ruler and protractor to construct triangles from written descriptions</li> <li>• Use ruler and compasses to construct triangles when all three sides known</li> </ul>
Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> <li>• Use a ruler to measure and draw lengths to the nearest millimetre</li> <li>• Use a protractor to measure and draw angles to the nearest degree</li> </ul>	Edge, Face, Vertex (Vertices) Plane Parallel Perpendicular Regular polygon Rotational symmetry  <b>Notation</b> The line between two points A and B is AB The angle made by points A, B and C is $\angle ABC$ The angle at the point A is $\hat{A}$ Arrow notation for sets of parallel lines Dash notation for sides of equal length	NCETM: <a href="#">Departmental workshop: Constructions</a> The equals sign was designed (by Robert Recorde in 1557) based on two equal length lines that are equidistant NCETM: <a href="#">Glossary</a>  <b>Common approaches</b> <i>Dynamic geometry software to be used by all students to construct and explore dynamic diagrams of perpendicular and parallel lines.</i>
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
<ul style="list-style-type: none"> <li>• Given SSS, how many different triangles can be constructed? Why? Repeat for ASA, SAS, SSA, AAS, AAA.</li> <li>• Always / Sometimes / Never: to draw a triangle you need to know the size of three angles; to draw a triangle you need to know the size of three sides.</li> <li>• Convince me that a hexagon can have rotational symmetry with order 2.</li> </ul>	KM: <a href="#">Shape work</a> (selected activities) KM: <a href="#">Rotational symmetry</a> NRICH: <a href="#">Notes on a triangle</a>  <b>Learning review</b> KM: <a href="#">7M13 BAM Task</a>	<ul style="list-style-type: none"> <li>• Two line segments that do not touch are perpendicular if they would meet at right angles when extended</li> <li>• Pupils may believe, incorrectly, that:                             <ul style="list-style-type: none"> <li>- perpendicular lines have to be horizontal / vertical</li> <li>- only straight lines can be parallel</li> <li>- all triangles have rotational symmetry of order 3</li> <li>- all polygons are regular</li> </ul> </li> </ul>



**Key concepts**

The Big Picture: [Properties of Shape progression map](#)

- identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres
- derive and apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; and triangles and other plane figures using appropriate language

[Return to overview](#)

Possible learning intentions		Possible success criteria
<ul style="list-style-type: none"> <li>• Investigate the properties of 3D shapes</li> <li>• Explore quadrilaterals</li> <li>• Explore triangles</li> </ul>		<ul style="list-style-type: none"> <li>• Know the vocabulary of 3D shapes</li> <li>• Know the connection between faces, edges and vertices in 3D shapes</li> <li>• Visualise a 3D shape from its net</li> <li>• Recall the names and shapes of special triangles and quadrilaterals</li> <li>• Know the meaning of a diagonal of a polygon</li> <li>• Know the properties of the special quadrilaterals (including diagonals)</li> <li>• Apply the properties of triangles to solve problems</li> <li>• Apply the properties of quadrilaterals to solve problems</li> </ul>
Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> <li>• Know the names of common 3D shapes</li> <li>• Know the meaning of face, edge, vertex</li> <li>• Understand the principle of a net</li> <li>• Know the names of special triangles</li> <li>• Know the names of special quadrilaterals</li> <li>• Know the meaning of parallel, perpendicular</li> <li>• Know the notation for equal sides, parallel sides, right angles</li> </ul> <p><b>Bring on the Maths<sup>+</sup>: Moving on up!</b> Properties of shapes: #1, #2</p>	<p>Face, Edge, Vertex (Vertices) Cube, Cuboid, Prism, Cylinder, Pyramid, Cone, Sphere Quadrilateral Square, Rectangle, Parallelogram, (Isosceles) Trapezium, Kite, Rhombus Delta, Arrowhead Diagonal Perpendicular Parallel Triangle Scalene, Right-angled, Isosceles, Equilateral</p> <p><b>Notation</b> Dash notation to represent equal lengths in shapes and geometric diagrams Right angle notation</p>	<p>Ensure that pupils do not use the word 'diamond' to describe a kite, or a square that is 45° to the horizontal. 'Diamond' is not the mathematical name of any shape. A cube is a special case of a cuboid and a rhombus is a special case of a parallelogram A prism must have a polygonal cross-section, and therefore a cylinder is not a prism. Similarly, a cone is not a pyramid. NCETM: <a href="#">Departmental workshop: 2D shapes</a> NCETM: <a href="#">Glossary</a></p> <p><b>Common approaches</b> <i>Every classroom has a set of <a href="#">triangle posters</a> and <a href="#">quadrilateral posters</a> on the wall</i> <i>Models of 3D shapes to be used by all students during this unit of work</i></p>
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
<ul style="list-style-type: none"> <li>• Show me an example of a trapezium. And another. And another ...</li> <li>• Always / Sometimes / Never: The number of vertices in a 3D shape is greater than the number of edges</li> <li>• Which quadrilaterals are special examples of other quadrilaterals? Why? Can you create a 'quadrilateral family tree'?</li> <li>• What is the same and what is different: Rhombus / Parallelogram?</li> </ul>	<p>KM: <a href="#">Euler's formula</a> KM: <a href="#">Visualising 3D shapes</a> KM: <a href="#">Dotty activities</a>: Shapes on dotted paper KM: <a href="#">What's special about quadrilaterals?</a> Constructing quadrilaterals from diagonals and summarising results. KM: <a href="#">Investigating polygons</a>. Tasks one and two should be carried out with irregular polygons. NRICH: <a href="#">Property chart</a> NRICH: <a href="#">Quadrilaterals game</a></p> <p><b>Learning review</b> <a href="http://www.diagnosticquestions.com">www.diagnosticquestions.com</a></p>	<ul style="list-style-type: none"> <li>• Some pupils may think that all trapezia are isosceles</li> <li>• Some pupils may think that a diagonal cannot be horizontal or vertical</li> <li>• Two line segments that do not touch are perpendicular if they would meet at right angles when extended. Therefore the diagonals of an arrowhead (delta) are perpendicular despite what some pupils may think</li> <li>• Some pupils may think that a square is only square if 'horizontal', and even that a 'non-horizontal' square is called a diamond</li> <li>• The equal angles of an isosceles triangle are not always the 'base angles' as some pupils may think</li> </ul>



## Key concepts

- understand and use the concepts and vocabulary of expressions, equations, formulae and terms
- use and interpret algebraic notation, including:  $ab$  in place of  $a \times b$ ,  $3y$  in place of  $y + y + y$  and  $3 \times y$ ,  $a^2$  in place of  $a \times a$ ,  $a^3$  in place of  $a \times a \times a$ ,  $a/b$  in place of  $a \div b$ , brackets
- simplify and manipulate algebraic expressions by collecting like terms and multiplying a single term over a bracket
- where appropriate, interpret simple expressions as functions with inputs and outputs
- substitute numerical values into formulae and expressions
- use conventional notation for priority of operations, including brackets

The Big Picture: [Algebra progression map](#)[Return to overview](#)

Possible learning intentions		Possible success criteria
<ul style="list-style-type: none"> <li>• Understand the vocabulary and notation of algebra</li> <li>• Manipulate algebraic expressions</li> <li>• Explore functions</li> <li>• Evaluate algebraic statements</li> </ul>		<ul style="list-style-type: none"> <li>• Know the meaning of expression, term, formula, equation, function</li> <li>• Know basic algebraic notation (the rules of algebra)</li> <li>• Use letters to represent variables</li> <li>• Identify like terms in an expression</li> <li>• Simplify an expression by collecting like terms</li> <li>• Know how to multiply a (positive) single term over a bracket (the distributive law)</li> <li>• Substitute positive numbers into expressions and formulae</li> <li>• Given a function, establish outputs from given inputs</li> <li>• Given a function, establish inputs from given outputs</li> <li>• Use a mapping diagram (function machine) to represent a function</li> <li>• Use an expression to represent a function</li> <li>• Use the order of operations correctly in algebraic situations</li> </ul>
Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> <li>• Use symbols (including letters) to represent missing numbers</li> <li>• Substitute numbers into worded formulae</li> <li>• Substitute numbers into simple algebraic formulae</li> <li>• Know the order of operations</li> </ul> <p><b>Bring on the Maths*: Moving on up!</b> Algebra: #1</p>	Algebra Expression, Term, Formula (formulae), Equation, Function, Variable Mapping diagram, Input, Output Represent Substitute Evaluate Like terms Simplify / Collect  <b>Notation</b> See key concepts above	Pupils will have experienced some algebraic ideas previously. Ensure that there is clarity about the distinction between representing a variable and representing an unknown. Note that each of the statements $4x$ , $42$ and $4\frac{1}{2}$ involves a different operation after the 4, and this can cause problems for some pupils when working with algebra. NCETM: <a href="#">Algebra</a> NCETM: <a href="#">Glossary</a>  <b>Common approaches</b> <i>All pupils are expected to learn about the connection between mapping diagrams and formulae (to represent functions) in preparation for future representations of functions graphically.</i>
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
<ul style="list-style-type: none"> <li>• Show me an example of an expression / formula / equation</li> <li>• Always / Sometimes / Never: <math>4(g+2) = 4g+8</math>, <math>3(d+1) = 3d+1</math>, <math>a^2 = 2a</math>, <math>ab = ba</math></li> <li>• <a href="#">What is wrong?</a></li> <li>• Jenny writes <math>2a + 3b + 5a - b = 7a + 3</math>. Kenny writes <math>2a + 3b + 5a - b = 9ab</math>. What would you write? Why?</li> </ul>	KM: <a href="#">Pairs in squares</a> . Prove the results algebraically. KM: <a href="#">Algebra rules</a> KM: Use <a href="#">number patterns</a> to develop the multiplying out of brackets KM: <a href="#">Algebra ordering cards</a> KM: <a href="#">Spiders and snakes</a> . See the 'clouding the picture' approach KM: <a href="#">Maths to Infinity: Brackets</a> NRICH: <a href="#">Your number is ...</a> NRICH: <a href="#">Crossed ends</a> NRICH: <a href="#">Number pyramids</a> and <a href="#">More number pyramids</a>  <b>Learning review</b> KM: <a href="#">7M7 BAM Task</a> , <a href="#">7M8 BAM Task</a> , <a href="#">7M9 BAM Task</a>	<ul style="list-style-type: none"> <li>• Some pupils may think that it is always true that <math>a=1</math>, <math>b=2</math>, <math>c=3</math>, etc.</li> <li>• A common misconception is to believe that <math>a^2 = a \times 2 = a2</math> or <math>2a</math> (which it can do on rare occasions but is not the case in general)</li> <li>• When working with an expression such as <math>5a</math>, some pupils may think that if <math>a=2</math>, then <math>5a = 52</math>.</li> <li>• Some pupils may think that <math>3(g+4) = 3g+4</math></li> <li>• The convention of not writing a coefficient of 1 (i.e. '1x' is written as 'x' may cause some confusion. In particular some pupils may think that <math>5h - h = 5</math></li> </ul>



**Key concepts**

- express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1
- define percentage as ‘number of parts per hundred’
- express one quantity as a percentage of another

The Big Picture: [Fractions, decimals and percentages progression map](#)

[Return to overview](#)

Possible learning intentions		Possible success criteria
<ul style="list-style-type: none"> <li>• Understand and use top-heavy fractions</li> <li>• Understand the meaning of ‘percentage’</li> <li>• Explore links between fractions and percentages</li> </ul>		<ul style="list-style-type: none"> <li>• Write one quantity as a fraction of another where the fraction is less than 1</li> <li>• Write one quantity as a fraction of another where the fraction is greater than 1</li> <li>• Write a fraction in its lowest terms by cancelling common factors</li> <li>• Convert between mixed numbers and top-heavy fractions</li> <li>• Understand that a percentage means ‘number of parts per hundred’</li> <li>• Write a percentage as a fraction</li> <li>• Write a quantity as a percentage of another</li> </ul>
Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> <li>• Understand the concept of a fraction as a proportion</li> <li>• Understand the concept of equivalent fractions</li> <li>• Understand the concept of equivalence between fractions and percentages</li> </ul> <p><b>Bring on the Maths*: Moving on up!</b> Fractions, decimals &amp; percentages: #1, #2</p>	Fraction Improper fraction Proper fraction Vulgar fraction Top-heavy fraction Percentage Proportion  <b>Notation</b> Diagonal fraction bar / horizontal fraction bar	Describe $\frac{1}{3}$ as ‘there are three equal parts and I take one’, and $\frac{3}{4}$ as ‘there are four equal parts and I take three’. Be alert to pupils reinforcing misconceptions through language such as ‘the bigger half’. To explore the equivalency of fractions make several copies of a diagram with three-quarters shaded. Show that splitting these diagrams with varying numbers of lines does not alter the fraction of the shape that is shaded. NRICH: <a href="#">Teaching fractions with understanding</a> NCETM: <a href="#">Teaching fractions</a> NCETM: <a href="#">Departmental workshop: Fractions</a> NCETM: <a href="#">Glossary</a>  <b>Common approaches</b> All pupils are made aware that ‘per cent’ is derived from Latin and means ‘out of one hundred’
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
<ul style="list-style-type: none"> <li>• Jenny says ‘<math>\frac{1}{10}</math> is the same as proportion as 10% so <math>\frac{1}{5}</math> is the same proportion as 5%.’ What do you think? Why?</li> <li>• What is the same and what is different: <math>\frac{1}{10}</math> and 10% ... <math>\frac{1}{5}</math> and 20%?</li> <li>• Show this fraction as part of a square / rectangle / number line / ...</li> </ul>	KM: <a href="#">Crazy cancelling, silly simplifying</a> NRICH: <a href="#">Rod fractions</a>  <b>Learning review</b> KM: <a href="#">7M3 BAM Task</a>	<ul style="list-style-type: none"> <li>• A fraction can be visualised as divisions of a shape (especially a circle) but some pupils may not recognise that these divisions must be equal in size, or that they can be divisions of any shape.</li> <li>• Pupils may not make the connection that a percentage is a different way of describing a proportion</li> <li>• Pupils may think that it is not possible to have a percentage greater than 100%</li> </ul>





<b>Key concepts</b>	<b>The Big Picture:</b> <a href="#">Ratio and Proportion progression map</a>
<ul style="list-style-type: none"> <li>use ratio notation, including reduction to simplest form</li> <li>divide a given quantity into two parts in a given part:part or part:whole ratio</li> </ul>	

[Return to overview](#)

Possible learning intentions	Possible success criteria	
<ul style="list-style-type: none"> <li>Understand and use ratio notation</li> <li>Solve problems that involve dividing in a ratio</li> </ul>	<ul style="list-style-type: none"> <li>Describe a comparison of measurements or objects using the language 'a to b'</li> <li>Describe a comparison of measurements or objects using ratio notation a:b</li> <li>Use ratio notation to describe a comparison of more than two measurements or objects</li> <li>Convert between different units of measurement</li> <li>State a ratio of measurements in the same units</li> <li>Simplify a ratio by cancelling common factors</li> <li>Identify when a ratio is written in its lowest terms</li> <li>Find the value of a 'unit' in a division in a ratio problem</li> <li>Divide a quantity in two parts in a given part:part ratio</li> <li>Divide a quantity in two parts in a given part:whole ratio</li> <li>Express correctly the solution to a division in a ratio problem</li> </ul>	
Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> <li>Find common factors of pairs of numbers</li> <li>Convert between standard metric units of measurement</li> <li>Convert between units of time</li> <li>Recall multiplication facts for multiplication tables up to <math>12 \times 12</math></li> <li>Recall division facts for multiplication tables up to <math>12 \times 12</math></li> <li>Solve comparison problems</li> </ul> <p><b>Bring on the Maths': Moving on!</b> Ratio and proportion: #1</p>	Ratio Proportion Compare, comparison Part Simplify Common factor Cancel Lowest terms Unit  <b>Notation</b> Ratio notation a:b for part:part or part:whole	Note that ratio notation is first introduced in this stage. When solving division in a ratio problems, ensure that pupils express their solution as two quantities rather than as a ratio. NCETM: <a href="#">The Bar Model</a> NCETM: <a href="#">Multiplicative reasoning</a> NCETM: <a href="#">Glossary</a>  <b>Common approaches</b> <i>All pupils are explicitly taught to use the bar model as a way to represent a division in a ratio problem</i>
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
<ul style="list-style-type: none"> <li>Show me a set of objects that demonstrates the ratio 3:2. And another, and another ...</li> <li>Convince me that the ratio 120mm:0.3m is equivalent to 2:5</li> <li>Always / Sometimes / Never: the smaller number comes first when writing a ratio</li> <li>Using Cuisenaire rods: If the red rod is 1, explain why d (dark green) is 3. Can you say the value for all the rods? (w, r, g, p, y, d, b, t, B, o). Extend this understanding of proportion by changing the unit rod e.g. if <math>r = 1</math>, <math>p = ?</math>; <math>b = ?</math>; <math>o + 2B = ?</math> If <math>B = 1</math>; <math>y = ?</math> <math>3y = ?</math>; <math>o = ?</math> <math>o + p = ?</math> if <math>o + r = 6/7</math>; <math>t = ?</math></li> </ul>	KM: <a href="#">Division in a ratio</a> and <a href="#">checking spreadsheet</a> KM: <a href="#">Maths to Infinity: FDP RP</a> KM: <a href="#">Stick on the Maths: Ratio and proportion</a> NRICH: <a href="#">Toad in the hole</a> NRICH: <a href="#">Mixing lemonade</a> NRICH: <a href="#">Food chains</a> NRICH: <a href="#">Tray bake</a>  <b>Learning review</b> <a href="http://www.diagnosticquestions.com">www.diagnosticquestions.com</a>	<ul style="list-style-type: none"> <li>Some pupils may think that a:b always means part:part</li> <li>Some pupils may try to simplify a ratio without first ensuring that the units of each part are the same</li> <li>Many pupils will want to identify an additive relationship between two quantities that are in proportion and apply this to other quantities in order to find missing amounts</li> </ul>



**Key concepts** The Big Picture: [Algebra progression map](#)

- generate terms of a sequence from a term-to-term rule

[Return to overview](#)

Possible learning intentions	Possible success criteria
<ul style="list-style-type: none"> <li>Explore number sequences</li> <li>Explore sequences</li> </ul>	<ul style="list-style-type: none"> <li>Use a term-to-term rule to generate a linear sequence</li> <li>Use a term-to-term rule to generate a non-linear sequence</li> <li>Find the term-to-term rule for a sequence</li> <li>Describe a number sequence</li> <li>Solve problems involving the term-to-term rule for a sequence</li> <li>Solve problems involving the term-to-term rule for a non-numerical sequence</li> </ul>

Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> <li>Know the vocabulary of sequences</li> <li>Find the next term in a linear sequence</li> <li>Find a missing term in a linear sequence</li> <li>Generate a linear sequence from its description</li> </ul> <p><b>Bring on the Maths<sup>+</sup>: Moving on up!</b> Number and Place Value: #4, #5</p>	Pattern Sequence Linear Term Term-to-term rule Ascending Descending	<p>'Term-to-term rule' is the only new vocabulary for this unit. Position-to-term rule, and the use of the nth term, are not developed until later stages.</p> <p>NRICH: <a href="#">Go forth and generalise</a>            NCETM: <a href="#">Algebra</a></p> <p><b>Common approaches</b>  <i>All students are taught to describe the term-to-term rule for both numerical and non-numerical sequences</i></p>

Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
<ul style="list-style-type: none"> <li>Show me a (non-)linear sequence. And another. And another.</li> <li>What's the same, what's different: 2, 5, 8, 11, 14, ... and 4, 7, 10, 13, 16, ...?</li> <li>Create a (non-linear/linear) sequence with a 3<sup>rd</sup> term of '7'</li> <li>Always/ Sometimes /Never: The 10<sup>th</sup> term of is double the 5<sup>th</sup> term of the (linear) sequence</li> <li>Kenny thinks that the 20<sup>th</sup> term of the sequence 5, 9, 13, 17, 21, ... will be 105. Do you agree with Kenny? Explain your answer.</li> </ul>	KM: <a href="#">Maths to Infinity: Sequences</a> KM: <a href="#">Growing patterns</a> NRICH: <a href="#">Shifting times tables</a> NRICH: <a href="#">Odds and evens and more evens</a>  <b>Learning review</b> <a href="http://www.diagnosticquestions.com">www.diagnosticquestions.com</a>	<ul style="list-style-type: none"> <li>When describing a number sequence some students may not appreciate the fact that the starting number is required as well as a term-to-term rule</li> <li>Some pupils may think that all sequences are ascending</li> <li>Some pupils may think the (2n)<sup>th</sup> term of a sequence is double the n<sup>th</sup> term of a (linear) sequence</li> </ul>



**Key concepts**

The Big Picture: [Measurement and mensuration progression map](#)

- use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)
- use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
- change freely between related standard units (e.g. time, length, area, volume/capacity, mass) in numerical contexts
- measure line segments and angles in geometric figures

[Return to overview](#)

Possible learning intentions		Possible success criteria	
<ul style="list-style-type: none"> <li>• Measure accurately</li> <li>• Convert between measures</li> <li>• Solve problems involving measurement</li> </ul>		<ul style="list-style-type: none"> <li>• Use a ruler to accurately measure line segments to the nearest millimetre</li> <li>• Use a protractor to accurately measure angles to the nearest degree</li> <li>• Convert fluently between metric units of length</li> <li>• Convert fluently between metric units of mass</li> <li>• Convert fluently between metric units of volume / capacity</li> <li>• Convert fluently between units of time</li> <li>• Convert fluently between units of money</li> <li>• Solve practical problems that involve converting between units</li> <li>• State conclusions clearly using the units correctly</li> </ul>	
Prerequisites	Mathematical language	Pedagogical notes	
<ul style="list-style-type: none"> <li>• Convert between metric units</li> <li>• Use decimal notation up to three decimal places when converting metric units</li> <li>• Convert between common Imperial units; e.g. feet and inches, pounds and ounces, pints and gallons</li> <li>• Convert between units of time</li> <li>• Use 12- and 24-hour clocks, both analogue and digital</li> </ul> <p><b>Bring on the Maths<sup>+</sup>: Moving on up!</b> Measures: #3</p>	Length, distance Mass, weight Volume Capacity Metre, centimetre, millimetre Tonne, kilogram, gram, milligram Litre, millilitre Hour, minute, second Inch, foot, yard Pound, ounce Pint, gallon Line segment  <b>Notation</b> Abbreviations of units in the metric system: m, cm, mm, kg, g, l, ml Abbreviations of units in the Imperial system: lb, oz	Weight and mass are distinct though they are often confused in everyday language. Weight is the force due to gravity, and is calculated as mass multiplied by the acceleration due to gravity. Therefore weight varies due to location while mass is a constant measurement. The prefix ‘centi-’ means one hundredth, and the prefix ‘milli-’ means one thousandth. These words are of Latin origin. The prefix ‘kilo-’ means one thousand. This is Greek in origin. Classify/Estimate angle first NCETM: <a href="#">Glossary</a>  <b>Common approaches</b> <i>Every classroom has a sack of sand (25 kg), a bag of sugar (1 kg), a cheque book (1 cheque is 1 gram), a bottle of water (1 litre, and also 1 kg of water) and a teaspoon (5 ml)</i>	
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions	
<ul style="list-style-type: none"> <li>• Show me another way of describing 2.5km. And another. And another.</li> <li>• Show me another way of describing 3.4 litres. And another. And another.</li> <li>• Show me another way of describing 3.7kg. And another. And another.</li> <li>• Kenny thinks that 14:30 is the same time as 2.30 p.m. Do you agree with Kenny? Explain your answer.</li> <li>• What’s the same, what’s different: 2 hours 30 minutes, 2.5 hours, 2½ hours and 2 hours 20 minutes?</li> </ul>	KM: <a href="#">Sorting units</a> KM: <a href="#">Another length</a> KM: <a href="#">Measuring space</a> KM: <a href="#">Another capacity</a> KM: <a href="#">Stick on the Maths: Units</a> NRICH: <a href="#">Temperature</a>  <b>Learning review</b> <a href="http://www.diagnosticquestions.com">www.diagnosticquestions.com</a>	<ul style="list-style-type: none"> <li>• Some pupils may write amounts of money incorrectly; e.g. £3.5 for £3.50, especially if a calculator is used at any point</li> <li>• Some pupils may apply an incorrect understanding that there are 100 minutes in a hour when solving problems</li> <li>• Some pupils may struggle when converting between 12- and 24-hour clock notation; e.g. thinking that 15:00 is 5 o’ clock</li> <li>• Some pupils may use the wrong scale of a protractor. For example, they measure an obtuse angle as 60° rather than 120°.</li> </ul>	



<b>Key concepts</b>	<b>The Big Picture:</b> <a href="#">Position and direction progression map</a>
<ul style="list-style-type: none"> <li>apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles</li> </ul>	

[Return to overview](#)

Possible learning intentions	Possible success criteria
<ul style="list-style-type: none"> <li>Investigate angles</li> </ul> <p><b>Bring on the Maths*:</b> Moving on up! Properties of shapes: #5</p>	<ul style="list-style-type: none"> <li>Identify fluently angles at a point, angles at a point on a line and vertically opposite angles</li> <li>Identify known angle facts in more complex geometrical diagrams</li> <li>Use knowledge of angles to calculate missing angles in geometrical diagrams</li> <li>Know that angles in a triangles total <math>180^\circ</math></li> <li>Find missing angles in triangles</li> <li>Find missing angles in isosceles triangles</li> <li>Explain reasoning using vocabulary of angles</li> </ul>

Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> <li>Identify angles that meet at a point</li> <li>Identify angles that meet at a point on a line</li> <li>Identify vertically opposite angles</li> <li>Know that vertically opposite angles are equal</li> </ul>	<p>Angle Degrees Right angle Acute angle Obtuse angle Reflex angle Protractor Vertically opposite Geometry, geometrical</p> <p><b>Notation</b> Right angle notation Arc notation for all other angles The degree symbol (<math>^\circ</math>)</p>	<p>It is important to make the connection between the total of the angles in a triangle and the sum of angles on a straight line by encouraging pupils to draw any triangle, rip off the corners of triangles and fitting them together on a straight line. However, this is not a proof and this needs to be revisited in Stage 8 using alternate angles to prove the sum is always <math>180^\circ</math>.</p> <p>The word 'isosceles' means 'equal legs'. What do you have at the bottom of equal legs? Equal ankles! NCETM: <a href="#">Glossary</a></p> <p><b>Common approaches</b> <i>Teachers convince pupils that the sum of the angles in a triangle is <math>180^\circ</math> by ripping the corners of triangles and fitting them together on a straight line.</i></p>

Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
<ul style="list-style-type: none"> <li>Show me possible values for a and b. And another. And another.</li> <li>Convince me that the angles in a triangle total <math>180^\circ</math></li> <li>Convince me that the angles in a quadrilateral must total <math>360^\circ</math></li> <li>What's the same, what's different: Vertically opposite angles, angles at a point, angles on a straight line and angles in a triangle?</li> <li>Kenny thinks that a triangle cannot have two obtuse angles. Do you agree? Explain your answer.</li> <li>Jenny thinks that the largest angle in a triangle is a right angle? Do you agree? Explain your thinking.</li> </ul> 	<p>KM: <a href="#">Maths to Infinity: Lines and angles</a> KM: <a href="#">Stick on the Maths: Angles</a> NRICH: <a href="#">Triangle problem</a> NRICH: <a href="#">Square problem</a> NRICH: <a href="#">Two triangle problem</a></p> <p><b>Learning review</b> <a href="http://www.diagnosticquestions.com">www.diagnosticquestions.com</a></p>	<ul style="list-style-type: none"> <li>Some pupils may think it's the 'base' angles of an isosceles that are always equal. For example, they may think that <math>a = b</math> rather than <math>a = c</math>.</li> </ul>  <ul style="list-style-type: none"> <li>Some pupils may make conceptual mistakes when adding and subtracting mentally. For example, they may see that one of two angles on a straight line is <math>127^\circ</math> and quickly respond that the other angle must be <math>63^\circ</math>.</li> </ul>



**Key concepts**

The Big Picture: [Fractions, decimals and percentages progression map](#)

- apply the four operations, including formal written methods, to simple fractions (proper and improper), and mixed numbers
- interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively
- compare two quantities using percentages
- solve problems involving percentage change, including percentage increase/decrease

[Return to overview](#)

**Possible learning intentions**

- Calculate with fractions
- Calculate with percentages

**Possible success criteria**

- Apply addition to proper fractions, improper fractions and mixed numbers
- Apply subtraction to proper fractions, improper fractions and mixed numbers
- Multiply proper and improper fractions
- Multiply mixed numbers
- Divide a proper fraction by a proper fraction
- Apply division to improper fractions and mixed numbers
- Use calculators to find a percentage of an amount using multiplicative methods
- Identify the multiplier for a percentage increase or decrease
- Use calculators to increase (decrease) an amount by a percentage using multiplicative methods
- Compare two quantities using percentages
- Know that percentage change = actual change ÷ original amount
- Calculate the percentage change in a given situation, including percentage increase / decrease

**Prerequisites**

- Add and subtract fractions with different denominators
- Add and subtract mixed numbers with different denominators
- Multiply a proper fraction by a proper fraction
- Divide a proper fraction by a whole number
- Simplify the answer to a calculation when appropriate
- Use non-calculator methods to find a percentage of an amount
- Convert between fractions, decimals and percentages

**Bring on the Maths<sup>+</sup>: Moving on up!**

Fractions, decimals & percentages: #3, #4, #5  
Ratio and proportion: #2

**Mathematical language**

Mixed number  
Equivalent fraction  
Simplify, cancel, lowest terms  
Proper fraction, improper fraction, top-heavy fraction, vulgar fraction  
Percent, percentage  
Multiplier  
Increase, decrease

**Notation**

Mixed number notation  
Horizontal / diagonal bar for fractions

**Pedagogical notes**

It is important that pupils are clear that the methods for addition and subtraction of fractions are different to the methods for multiplication and subtraction. A fraction wall is useful to help visualise and re-present the calculations.

NCETM: [The Bar Model](#), [Teaching fractions](#), [Fractions videos](#)  
NCETM: [Glossary](#)

**Common approaches**

*When multiplying a decimal by a whole number pupils are taught to use the corresponding whole number calculation as a general strategy*  
*When adding and subtracting mixed numbers pupils are taught to convert to improper fractions as a general strategy*  
*Teachers use the horizontal fraction bar notation at all times*

**Reasoning opportunities and probing questions**

- Show me a proper (improper) fraction. And another. And another.
- Show me a mixed number fraction. And another. And another.
- Jenny thinks that you can only multiply fractions if they have the same common denominator. Do you agree with Jenny? Explain your answer.
- Benny thinks that you can only divide fractions if they have the same common denominator. Do you agree with Jenny? Explain.
- Kenny thinks that  $\frac{6}{10} \div \frac{3}{2} = \frac{2}{5}$ . Do you agree with Kenny? Explain.
- Always/Sometimes/Never: To reverse an increase of x%, you decrease by x%
- Lenny calculates the % increase of £6 to £8 as 25%. Do you agree with Lenny? Explain your answer.

**Suggested activities**

KM: [Stick on the Maths: Percentage increases and decreases](#)  
KM: [Maths to Infinity: FDP RP](#)  
KM: [Percentage methods](#)  
KM: [Mixed numbers: mixed approaches](#)  
NRICH: [Would you rather?](#)  
NRICH: [Keep it simple](#)  
NRICH: [Egyptian fractions](#)  
NRICH: [The greedy algorithm](#)  
NRICH: [Fractions jigsaw](#)  
NRICH: [Countdown fractions](#)

**Learning review**

KM: [7M4 BAM Task](#), [7M5 BAM Task](#)

**Possible misconceptions**

- Some pupils may think that you simply can simply add/subtract the whole number part of mixed numbers and add/subtract the fractional part of mixed numbers when adding/subtracting mixed numbers, e.g.  $3\frac{1}{3} - 2\frac{1}{2} = 1\frac{-1}{6}$
- Some pupils may make multiplying fractions over complicated by applying the same process for adding and subtracting of finding common denominators.
- Some pupils may think the multiplier for, say, a 20% decrease is 0.2 rather than 0.8
- Some pupils may think that percentage change = actual change ÷ new amount



**Key concepts**

The Big Picture: [Algebra progression map](#)

- recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions)
- solve linear equations in one unknown algebraically

[Return to overview](#)

Possible learning intentions	Possible success criteria
<ul style="list-style-type: none"> <li>• Explore way of solving equations</li> <li>• Solve two-step equations</li> <li>• Solve three-step equations</li> </ul>	<ul style="list-style-type: none"> <li>• Choose the required inverse operation when solving an equation</li> <li>• Identify the correct order of undoing the operations in an equation</li> <li>• Solve one-step equations when the solution is a whole number (fraction)</li> <li>• Solve two-step equations (including the use of brackets) when the solution is a whole number</li> <li>• Solve two-step equations (including the use of brackets) when the solution is a fraction</li> <li>• Solve three-step equations (including the use of brackets) when the solution is a whole number</li> <li>• Solve three-step equations (including the use of brackets) when the solution is a fraction</li> <li>• Check the solution to an equation by substitution</li> </ul>

Prerequisites	Mathematical language	Pedagogical notes												
<ul style="list-style-type: none"> <li>• Know the basic rules of algebraic notation</li> <li>• Express missing number problems algebraically</li> <li>• Solve missing number problems expressed algebraically</li> </ul> <p><b>Bring on the Maths<sup>+</sup>: Moving on up!</b> Algebra: #2</p>	<p>Algebra, algebraic, algebraically Unknown Equation Operation Solve Solution Brackets Symbol Substitute</p> <p><b>Notation</b> The lower case and upper case of a letter should not be used interchangeably when worked with algebra Juxtaposition is used in place of 'x'. 2a is used rather than a2. Division is written as a fraction</p>	<p>This unit focuses on solving linear equations with unknowns on one side. Although linear equations with the unknown on both sides are addressed in Stage 8, pupils should be encouraged to think how to solve these equations by exploring the equivalent family of equations such as if <math>2x = 8</math> then <math>2x + 2 = 10</math>, <math>2x - 3 = 5</math>, <math>3x = x + 8</math>, <math>3x + 2 = x + 10</math>, etc.</p> <p>Encourage pupils to re-present the equations such as <math>2x + 8 = 23</math> using the Bar Model.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>x</td> <td>8</td> </tr> <tr> <td colspan="3" style="text-align: center;">23</td> </tr> </table> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>x</td> </tr> <tr> <td colspan="2" style="text-align: center;">15</td> </tr> </table> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> </tr> <tr> <td style="text-align: center;">7.5</td> </tr> </table> <p>NCETM: <a href="#">The Bar Model</a> NCETM: <a href="#">Algebra</a>, NCETM: <a href="#">Glossary</a></p> <p><b>Common approaches</b> <i>Pupils should explore solving equations by applying inverse operations, but the expectation is that all pupils should solve by balancing:</i></p> $\begin{array}{r} 2x + 8 = 23 \\ - 8 \quad - 8 \\ \hline 2x = 15 \\ \div 2 \quad 2 \\ \hline x = 7.5 \text{ (or } 15/2) \end{array}$ <p><i>Pupils are expected to multiply out the brackets before solving an equation involving brackets. This makes the connection with two step equations such as <math>2x + 6 = 22</math></i></p>	x	x	8	23			x	x	15		x	7.5
x	x	8												
23														
x	x													
15														
x														
7.5														

Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions		
<ul style="list-style-type: none"> <li>• Show me an (one-step, two-step) equation with a solution of 14 (positive, fractional solution). And another. And another ...</li> <li>• Kenny thinks if <math>6x = 3</math> then <math>x = 2</math>. Do you agree with Kenny? Explain</li> <li>• Jenny and Lenny are solving: <math>3(x - 2) = 51</math>. Who is correct? Explain</li> </ul> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; vertical-align: top;"> <p><i>Jenny's solution</i></p> <math display="block">\begin{array}{r} 3(x - 2) = 15 \\ \div 3 \quad \div 3 \\ x - 2 = 5 \\ \div 2 \quad \div 2 \\ x = 7 \end{array}</math> </td> <td style="width: 50%; vertical-align: top;"> <p><i>Lenny's solution</i></p> <math display="block">\begin{array}{r} 3(x - 2) = 15 \\ \text{Multiplying out brackets} \\ 3x - 6 = 15 \\ +2 \quad +2 \\ 3x = 21 \\ \div 3 \quad \div 3 \\ x = 7 \end{array}</math> </td> </tr> </table>	<p><i>Jenny's solution</i></p> $\begin{array}{r} 3(x - 2) = 15 \\ \div 3 \quad \div 3 \\ x - 2 = 5 \\ \div 2 \quad \div 2 \\ x = 7 \end{array}$	<p><i>Lenny's solution</i></p> $\begin{array}{r} 3(x - 2) = 15 \\ \text{Multiplying out brackets} \\ 3x - 6 = 15 \\ +2 \quad +2 \\ 3x = 21 \\ \div 3 \quad \div 3 \\ x = 7 \end{array}$	<p>KM: <a href="#">Spiders and snakes</a>. The example is for an unknown on both sides but the same idea can be used. NRICH: <a href="#">Inspector Remorse</a> NRICH: <a href="#">Quince, quonce, quance</a> NRICH: <a href="#">Weighing the baby</a></p> <p><b>Learning review</b> KM: <a href="#">7M10 BAM Task</a></p>	<ul style="list-style-type: none"> <li>• Some pupils may think that equations always need to be presented in the form <math>ax + b = c</math> rather than <math>c = ax + b</math>.</li> <li>• Some pupils may think that the solution to an equation is always positive and/or a whole number.</li> <li>• Some pupils may get the use the inverse operations in the wrong order, for example, to solve <math>2x + 18 = 38</math> the pupils divide by 2 first and then subtract 18.</li> </ul>
<p><i>Jenny's solution</i></p> $\begin{array}{r} 3(x - 2) = 15 \\ \div 3 \quad \div 3 \\ x - 2 = 5 \\ \div 2 \quad \div 2 \\ x = 7 \end{array}$	<p><i>Lenny's solution</i></p> $\begin{array}{r} 3(x - 2) = 15 \\ \text{Multiplying out brackets} \\ 3x - 6 = 15 \\ +2 \quad +2 \\ 3x = 21 \\ \div 3 \quad \div 3 \\ x = 7 \end{array}$			




**Key concepts**

- use standard units of measure and related concepts (length, area, volume/capacity)
- calculate perimeters of 2D shapes
- know and apply formulae to calculate area of triangles, parallelograms, trapezia
- calculate surface area of cuboids
- know and apply formulae to calculate volume of cuboids
- understand and use standard mathematical formulae

The Big Picture: [Measurement and mensuration progression map](#)

[Return to overview](#)

Possible learning intentions		Possible success criteria	
<ul style="list-style-type: none"> <li>• Develop knowledge of area</li> <li>• Investigate surface area</li> <li>• Explore volume</li> </ul>		<ul style="list-style-type: none"> <li>• Recognise that the value of the perimeter can equal the value of area</li> <li>• Use standard formulae for area and volume</li> <li>• Find missing lengths in 2D shapes when the area is known</li> <li>• Know that the area of a trapezium is given by the formula <math>\text{area} = \frac{1}{2} \times (a + b) \times h = \left(\frac{a+b}{2}\right) h = \frac{(a+b)h}{2}</math></li> <li>• Calculate the area of a trapezium</li> <li>• Understand the meaning of surface area</li> <li>• Find the surface area of cuboids (including cubes) when lengths are known</li> <li>• Find missing lengths in 3D shapes when the volume or surface area is known</li> </ul>	
Prerequisites	Mathematical language	Pedagogical notes	
<ul style="list-style-type: none"> <li>• Understand the meaning of area, perimeter, volume and capacity</li> <li>• Know how to calculate areas of rectangles, parallelograms and triangles using the standard formulae</li> <li>• Know that the area of a triangle is given by the formula <math>\text{area} = \frac{1}{2} \times \text{base} \times \text{height} = \text{base} \times \text{height} \div 2 = \frac{bh}{2}</math></li> <li>• Know appropriate metric units for measuring area and volume</li> </ul> <p><b>Bring on the Maths<sup>+</sup>: Moving on up!</b> Measures: #4, #5, #6</p>	Perimeter, area, volume, capacity, surface area Square, rectangle, parallelogram, triangle, trapezium (trapezia) Polygon Cube, cuboid Square millimetre, square centimetre, square metre, square kilometre Cubic centimetre, centimetre cube Formula, formulae Length, breadth, depth, height, width  <b>Notation</b> Abbreviations of units in the metric system: km, m, cm, mm, mm <sup>2</sup> , cm <sup>2</sup> , m <sup>2</sup> , km <sup>2</sup> , mm <sup>3</sup> , cm <sup>3</sup> , km <sup>3</sup>	Ensure that pupils make connections with the area and volume work in Stage 6 and below, in particular the importance of the perpendicular height. NCETM: <a href="#">Glossary</a>  <b>Common approaches</b> <i>Pupils have already derived the formula for the area of a parallelogram. They use this to derive the formula for the area of a trapezium as <math>\frac{(a+b)h}{2}</math> by copying and rotating a trapezium as shown above.</i>  <i>Pupils use the area of a triangle as given by the formula <math>\text{area} = \frac{bh}{2}</math>.</i> Every classroom has a set of <a href="#">area posters</a> on the wall.	
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions	
<ul style="list-style-type: none"> <li>• Always / Sometimes / Never: The value of the volume of a cuboid is greater than the value of the surface area</li> <li>• Convince me that the area of a triangle = <math>\frac{1}{2} \times \text{base} \times \text{height} = \text{base} \times \text{height} \div 2 = \frac{bh}{2}</math></li> <li>• (Given a right-angled trapezium with base labelled 8 cm, height 5 cm, top 6 cm) Kenny uses the formula for the area of a trapezium and Benny splits the shape into a rectangle and a triangle. What would you do? Why?</li> </ul>	KM: <a href="#">Equable shapes</a> (for both 2D and 3D shapes) KM: <a href="#">Triangle takeaway</a> KM: <a href="#">Surface area</a> KM: <a href="#">Class of rice</a> KM: <a href="#">Stick on the Maths: Area and Volume</a> KM: <a href="#">Maths to Infinity: Area and Volume</a> NRICH: <a href="#">Can They Be Equal?</a>  <b>Learning review</b> KM: <a href="#">7M12 BAM Task</a>	<ul style="list-style-type: none"> <li>• Some pupils may use the sloping height when finding the areas of parallelograms, triangles and trapezia</li> <li>• Some pupils may think that the area of a triangle is found using <math>\text{area} = \text{base} \times \text{height}</math></li> <li>• Some pupils may think that you multiply all the numbers to find the area of a shape</li> <li>• Some pupils may confuse the concepts of surface area and volume</li> <li>• Some pupils may only find the area of the three 'distinct' faces when finding surface area</li> </ul>	



**Key concepts**

The Big Picture: [Number and Place Value progression map](#)

- round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures)
- estimate answers; check calculations using approximation and estimation, including answers obtained using technology
- recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions)

[Return to overview](#)

Possible learning intentions		Possible success criteria
<ul style="list-style-type: none"> <li>• Explore ways of approximating numbers</li> <li>• Explore ways of checking answers</li> </ul>		<ul style="list-style-type: none"> <li>• Approximate by rounding to any number of decimal places</li> <li>• Know how to identify the first significant figure in any number</li> <li>• Approximate by rounding to the first significant figure in any number</li> <li>• Understand estimating as the process of finding a rough value of an answer or calculation</li> <li>• Use estimation to predict the order of magnitude of the solution to a (decimal) calculation</li> <li>• Estimate calculations by rounding numbers to one significant figure</li> <li>• Use cancellation to simplify calculations</li> <li>• Use inverse operations to check solutions to calculations</li> </ul>
Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> <li>• Approximate any number by rounding to the nearest 10, 100 or 1000, 10 000, 100 000 or 1 000 000</li> <li>• Approximate any number with one or two decimal places by rounding to the nearest whole number</li> <li>• Approximate any number with two decimal places by rounding to the one decimal place</li> <li>• Simplify a fraction by cancelling common factors</li> </ul>	Approximate (noun and verb) Round Decimal place Check Solution Answer Estimate (noun and verb) Order of magnitude Accurate, Accuracy Significant figure Cancel Inverse Operation  <b>Notation</b> The approximately equal symbol ( $\approx$ ) Significant figure is abbreviated to 's.f.' or 'sig fig'	This unit is an opportunity to develop and practice calculation skills with a particular emphasis on checking, approximating or estimating the answer. Pupils should be able to estimate calculations involving integers and decimals. Also see big pictures: <a href="#">Calculation progression map</a> and <a href="#">Fractions, decimals and percentages progression map</a> NCETM: <a href="#">Glossary</a>  <b>Common approaches</b> <i>All pupils are taught to visualise rounding through the use a number line</i>
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
<ul style="list-style-type: none"> <li>• Convince me that 39 652 rounds to 40 000 to one significant figure</li> <li>• Convince me that 0.6427 does <u>not</u> round to 1 to one significant figure</li> <li>• What is wrong: <math>\frac{11 \times 28.2}{0.54} \approx \frac{10 \times 30}{0.5} = 150</math>. How can you correct it?</li> </ul>	KM: <a href="#">Approximating calculations</a> KM: <a href="#">Stick on the Maths: CALC6: Checking solutions</a>  <b>Learning review</b> KM: <a href="#">7M6 BAM Task</a>	<ul style="list-style-type: none"> <li>• Some pupils may truncate instead of round</li> <li>• Some pupils may round down at the half way point, rather than round up.</li> <li>• Some pupils may think that a number between 0 and 1 rounds to 0 or 1 to one significant figure</li> <li>• Some pupils may divide by 2 when the denominator of an estimated calculation is 0.5</li> </ul>





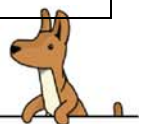
**Key concepts**

The Big Picture: [Position and direction progression map](#)

- work with coordinates in all four quadrants
- understand and use lines parallel to the axes,  $y = x$  and  $y = -x$
- solve geometrical problems on coordinate axes
- identify, describe and construct congruent shapes including on coordinate axes, by considering rotation, reflection and translation
- describe translations as 2D vectors

[Return to overview](#)

Possible learning intentions		Possible success criteria
<ul style="list-style-type: none"> <li>• Explore lines on the coordinate grid</li> <li>• Use transformations to move shapes</li> <li>• Describe transformations</li> </ul>		<ul style="list-style-type: none"> <li>• Write the equation of a line parallel to the x-axis or the y-axis</li> <li>• Draw a line parallel to the x-axis or the y-axis given its equation</li> <li>• Identify the lines <math>y = x</math> and <math>y = -x</math></li> <li>• Draw the lines <math>y = x</math> and <math>y = -x</math></li> <li>• Carry out a reflection in a diagonal mirror line (<math>45^\circ</math> from horizontal)</li> <li>• Find and name the equation of the mirror line for a given reflection</li> <li>• Describe a translation as a 2D vector</li> <li>• Understand the concept and language of rotations</li> <li>• Carry out a rotation using a given angle, direction and centre of rotation</li> <li>• Describe a rotation using mathematical language</li> </ul>
Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> <li>• Work with coordinates in all four quadrants</li> <li>• Carry out a reflection in a given vertical or horizontal mirror line</li> <li>• Carry out a translation</li> </ul> <p><b>Bring on the Maths<sup>®</sup>: Moving on up!</b> Position and direction: #1, #2</p>	(Cartesian) coordinates Axis, axes, x-axis, y-axis Origin Quadrant Translation, Reflection, Rotation Transformation Object, Image Congruent, congruence Mirror line Vector Centre of rotation  <b>Notation</b> Cartesian coordinates should be separated by a comma and enclosed in brackets (x, y) Vector notation $\begin{pmatrix} a \\ b \end{pmatrix}$ where a = movement right and b = movement up	Pupils should be able to use a centre of rotation that is outside, inside, or on the edge of the object Pupils should be encouraged to see the line $x = a$ as the complete (and infinite) set of points such that the x-coordinate is a. The French mathematician Rene Descartes introduced Cartesian coordinates in the 17 <sup>th</sup> century. It is said that he thought of the idea while watching a fly moving around on his bedroom ceiling. NCETM: <a href="#">Glossary</a>  <b>Common approaches</b> <i>Pupils use ICT to explore these transformations</i> <i>Teachers do not use the phrase 'along the corridor and up the stairs' as it can encourage a mentality of only working in the first quadrant. Later, pupils will have to use coordinates in all four quadrants. A more helpful way to remember the order of coordinates is 'x is a cross, wise up!'</i> <i>Teachers use the language 'negative number', and not 'minus number', to avoid future confusion with calculations.</i>
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
<ul style="list-style-type: none"> <li>• Always / Sometimes / Never: The centre of rotation is in the centre of the object</li> <li>• Convince me that <math>y = 0</math> is the x-axis</li> <li>• Always / Sometimes / Never: The line <math>x = a</math> is parallel to the x-axis</li> </ul>	KM: <a href="#">Lines</a> KM: <a href="#">Moving house</a> KM: <a href="#">Transformations: Bop It?</a> KM: Dynamic Autograph files: <a href="#">Reflection</a> , <a href="#">Rotation</a> , <a href="#">Translation</a> KM: <a href="#">Autograph transformations</a> KM: <a href="#">Stick on the Maths SSM7: Transformations</a> NRICH: <a href="#">Transformation Game</a>  <b>Learning review</b> KM: <a href="#">7M11 BAM Task</a>	<ul style="list-style-type: none"> <li>• Some pupils will wrestle with the idea that a line <math>x = a</math> is parallel to the y-axis</li> <li>• When describing or carrying out a translation, some pupils may count the squares between the two shapes rather than the squares that describe the movement between the two shapes.</li> <li>• When reflecting a shape in a diagonal mirror line some students may draw a translation</li> <li>• Some pupils may think that the centre of rotation is always in the centre of the shape</li> <li>• Some pupils will confuse the order of x- and y-coordinates</li> <li>• When constructing axes, some pupils may not realise the importance of equal divisions on the axes</li> </ul>



**Key concepts**

The Big Picture: [Statistics progression map](#)

- interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data and know their appropriate use

[Return to overview](#)

Possible learning intentions		Possible success criteria
<ul style="list-style-type: none"> <li>Explore types of data</li> <li>Construct and interpret graphs</li> <li>Select appropriate graphs and charts</li> </ul>		<ul style="list-style-type: none"> <li>Know the meaning of categorical data</li> <li>Know the meaning of discrete data</li> <li>Interpret and construct frequency tables</li> <li>Construct and interpret pictograms (bar charts, tables) and know their appropriate use</li> <li>Construct and interpret comparative bar charts</li> <li>Interpret pie charts and know their appropriate use</li> <li>Construct pie charts when the total frequency is not a factor of 360</li> <li>Choose appropriate graphs or charts to represent data</li> <li>Construct and interpret vertical line charts</li> </ul>
Prerequisites	Mathematical language	Pedagogical notes
<ul style="list-style-type: none"> <li>Construct and interpret a pictogram</li> <li>Construct and interpret a bar chart</li> <li>Construct and interpret a line graph</li> <li>Understand that pie charts are used to show proportions</li> <li>Use a template to construct a pie chart by scaling frequencies</li> </ul> <p><b>Bring on the Maths<sup>+</sup>: Moving on up!</b> Statistics: #1, #2, #3</p>	Data, Categorical data, Discrete data Pictogram, Symbol, Key Frequency Table, Frequency table Tally Bar chart Time graph, Time series Bar-line graph, Vertical line chart Scale, Graph Axis, axes Line graph Pie chart Sector Angle Maximum, minimum  <b>Notation</b> When tallying, groups of five are created by striking through each group of four	<p>In stage 6 pupils constructed pie charts when the total of frequencies is a factor of 360. More complex cases can now be introduced. Much of the content of this unit has been covered previously in different stages. This is an opportunity to bring together the full range of skills encountered up to this point, and to develop a more refined understanding of usage and vocabulary.</p> <p>William Playfair, a Scottish engineer and economist, introduced the bar chart and line graph in 1786. He also introduced the pie chart in 1801. NCETM: <a href="#">Glossary</a></p> <p><b>Common approaches</b>  <i>Pie charts are constructed by calculating the angle for each section by dividing 360 by the total frequency and not using percentages. The angle for the first section is measured from a vertical radius. Subsequent sections are measured using the boundary line of the previous section.</i></p>
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions
<ul style="list-style-type: none"> <li>Show me a pie chart representing the following information: Blue (30%), Red (50%), Yellow (the rest). And another. And another.</li> <li>Always / Sometimes / Never: Bar charts are vertical</li> <li>Always / Sometimes / Never: Bar charts, pie charts, pictograms and vertical line charts can be used to represent any data</li> <li>Kenny says 'If two pie charts have the same section then the amount of data the section represents is the same in each pie chart.' Do you agree with Kenny? Explain your answer.</li> </ul>	KM: <a href="#">Constructing pie charts</a> KM: <a href="#">Maths to Infinity: Averages, Charts and Tables</a> NRICH: <a href="#">Picturing the World</a> NRICH: <a href="#">Charting Success</a>  <b>Learning review</b> <a href="http://www.diagnosticquestions.com">www.diagnosticquestions.com</a>	<ul style="list-style-type: none"> <li>Some pupils may think that a line graph is appropriate for discrete data</li> <li>Some pupils may think that each square on the grid used represents one unit</li> <li>Some pupils may confuse the fact that the sections of the pie chart total 100% and 360°</li> <li>Some pupils may not leave gaps between the bars of a bar chart</li> </ul>



**Key concepts**

The Big Picture: [Statistics progression map](#)

- interpret, analyse and compare the distributions of data sets from univariate empirical distributions through appropriate measures of central tendency (median, mean and mode) and spread (range)

[Return to overview](#)

Possible learning intentions		Possible success criteria	
<ul style="list-style-type: none"> <li>Investigate averages</li> <li>Explore ways of summarising data</li> <li>Analyse and compare sets of data</li> </ul>		<ul style="list-style-type: none"> <li>Understand the mode and median as measures of typicality (or location)</li> <li>Find the mode of set of data</li> <li>Find the median of a set of data</li> <li>Find the median of a set of data when there are an even number of numbers in the data set</li> <li>Use the mean to find a missing number in a set of data</li> <li>Calculate the mean from a frequency table</li> <li>Find the mode from a frequency table</li> <li>Find the median from a frequency table</li> <li>Understand the range as a measure of spread (or consistency)</li> <li>Calculate the range of a set of data</li> <li>Analyse and compare sets of data</li> <li>Appreciate the limitations of different statistics (mean, median, mode, range)</li> </ul>	
Prerequisites	Mathematical language	Pedagogical notes	
<ul style="list-style-type: none"> <li>Understand the meaning of 'average' as a typicality (or location)</li> <li>Calculate the mean of a set of data</li> </ul> <p><b>Bring on the Maths<sup>4</sup>: Moving on up!</b> Statistics: #4</p>	Average Spread Consistency Mean Median Mode Range Measure Data Statistic Statistics Approximate Round	The word 'average' is often used synonymously with the mean, but it is only one type of average. In fact, there are several different types of mean (the one in this unit properly being named as the 'arithmetic mean'). NCETM: <a href="#">Glossary</a>  <b>Common approaches</b> Every classroom has a set of <a href="#">statistics posters</a> on the wall Always use brackets when writing out the calculation for a mean, e.g. $(2 + 3 + 4 + 5) \div 4 = 14 \div 4 = 3.5$	
Reasoning opportunities and probing questions	Suggested activities	Possible misconceptions	
<ul style="list-style-type: none"> <li>Show me a set of data with a mean (mode, median, range) of 5.</li> <li>Always / Sometimes / Never: The mean is greater than the mode for a set of data</li> <li>Always / Sometimes / Never: The mean is greater than the median for a set of data</li> <li>Convince me that a set of data could have more than one mode.</li> <li>What's the same and what's different: mean, mode, median, range?</li> </ul>	KM: <a href="#">Maths to Infinity: Averages</a> KM: <a href="#">Maths to Infinity: Averages, Charts and Tables</a> KM: <a href="#">Stick on the Maths HD4: Averages</a> NRICH: <a href="#">M, M and M</a> NRICH: <a href="#">The Wisdom of the Crowd</a>  <b>Learning review</b> <a href="http://www.diagnosticquestions.com">www.diagnosticquestions.com</a>	<ul style="list-style-type: none"> <li>If using a calculator some pupils may not use the '=' symbol (or brackets) correctly; e.g. working out the mean of 2, 3, 4 and 5 as <math>2 + 3 + 4 + 5 \div 4 = 10.25</math>.</li> <li>Some pupils may think that the range is a type of average</li> <li>Some pupils may think that a set of data with an even number of items has two values for the median, e.g. 2, 4, 5, 6, 7, 8 has a median of 5 and 6 rather than 5.5</li> <li>Some pupils may not write the data in order before finding the median.</li> </ul>	

